BRIEF PAPER

A Simplified Method for Determining Mathematical Representation of Microwave Oscillator Load Characteristics

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SUMMARY Previously a method was reported to determine the mathematical representation of the microwave oscillator admittance by using numerical calculation. When analyzing the load characteristics and synchronization phenomena by using this formula, the analysis results meet with the experimental results. This paper describes a method to determine the mathematical representation manually.

key words: *microwave oscillator, High frequency oscillator, van der Pol, Rieke diagram, mathematical representation*

1. Introduction

It is time consuming to measure and plot the overall load characteristics (Rieke diagram), but it has been used to express the characteristics of microwave oscillators, since the load characteristics can be represented within a circle of the finite extent. Previously a method was reported to determine the mathematical representation of the microwave oscillator admittance by using Rieke diagram and numerical calculation [1]–[6]. When analyzing the load characteristics (Rieke diagram) by using the oscillator admittance formula (1) that the voltage-dependent susceptance component (B_v) and frequency dependent components ($G_\omega, Y_{\omega 2}$) are added to the van der Pol oscillator model [6]–[9], the analysis results meet well with the experimental results [8]. The coefficients G_ω, $G_{\omega 2}$ make the constant power contour more practical than the van der Pol oscillator model on Rieke diagram, and B_v curves the constant frequency contours on Rieke diagram [6]. As for the mutual synchronization phenomena, $G_{\omega 2}$ makes unstable domain of anti-phase mode wider [10]. B_v well explains the injection-locking characteristics [8]. B_v also makes the anti-phase locking frequency a little bit different from in-phase locking frequency and makes anti-phase locking range asymmetric in shape in mutual synchronization characteristics [11]. The formula (2) does not have the frequency dependent components (G_ω, $Y_{\omega 2}$) but it has B_v. So, when $|B_v|$ becomes larger, the formula (2) reproduces the synchronization characteristics better than the van der Pol oscillator model [8], [11]–[16]. This paper describes a method to conveniently determine the mathematical representation not using numerical calculation but using manual calculation.

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2. Mathematical Expression of Oscillator Admittance

The mathematical representation of microwave oscillator admittance is shown as (1) and (2) [6], [8].

$$Y(j\omega, |V|^2) = -G_0 + jB_0 + (G_\omega + jB_\omega)\cdot \varDelta\omega + (G_v + jB_v)|V|^2 + Y_{\omega 2}\cdot \varDelta\omega^2 \quad (1)$$

Where, $\varDelta\omega = \omega - \omega_0$, $\omega_0/2\pi$ is the center frequency that is the oscillation frequency when the matched load ($G_L = Y_0$, $B_L = 0$) is connected to the line. Y_0 is the characteristic admittance of the line, and the coefficient $Y_{\omega 2} = G_{\omega 2} + jB_{\omega 2}$ is complex number.

$$Y(j\omega, |V|^2) = -G_0 + jB_0 + jB_\omega\cdot \varDelta\omega + (G_v + jB_v)|V|^2 \quad (2)$$

Now, let the load admittance be $Y_L = G_L + jB_L$, and connect Y_L to the oscillator admittance (2).

$$Y(j\omega, |V|^2) + Y_L = 0 \quad (3)$$

The real part of (3) and the output power of oscillator are

$$-G_0 + G_v|V|^2 + G_L = 0 \quad (4)$$
$$P = G_L|V|^2$$

From (4), the oscillator generates its maximum output power P_m, when $G_L = G_0/2$ (Fig. 1).

$$P_m = G_0^2/4G_v \quad (5)$$

Substitute (4) into the imaginary part of (3) to eliminate $|V|^2$. The constant frequency contour is

$$B_0 + B_L = (G_L - G_0)B_v/G_v - B_\omega\cdot \varDelta\omega \quad (6)$$

The coefficients ratio B_v/G_v is the slope (K) of the constant frequency contour ($\varDelta\omega/2\pi = 0$) (Fig. 2). When connecting

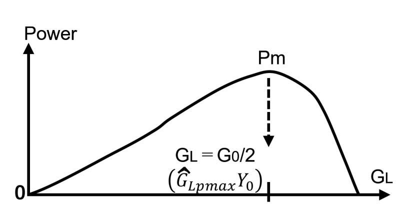

Fig. 1 Power versus load G_L ($B_L = 0$).

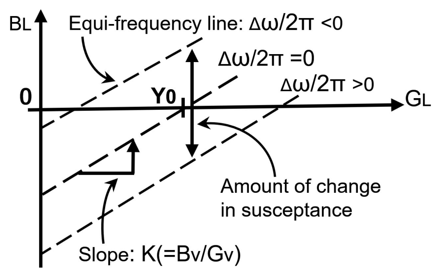

Fig. 2 Complex load plane (K > 0).

the matched load ($G_L = Y_0$, $B_L = 0$) to the oscillator, the coefficient B_0 is

$$B_0 = (Y_0 - G_0)B_v/G_v \tag{7}$$

As is shown in Fig. 2, putting $G_L = Y_0$, the coefficient B_ω can be determined by measuring the changes in frequency and load susceptance (Appendix).

The coefficients G_0, G_v and B_ω are usually positive, so the stability criteria $B_\omega G_v > 0$ are satisfied [6].

2.1 Normalized Formula of Oscillator Admittance

Let (2) be normalized by using (9) [8].

$$\hat{Y}(jx, |\hat{V}|^2) = -\hat{G}_0 + j\hat{B}_0 + jx + (\hat{G}_0/2 + j\hat{B}_v)|\hat{V}|^2 \tag{8}$$

$$\left.\begin{array}{l}\delta = (\omega - \omega_0)/\omega_0,\ Q = \omega_0 B_\omega/2Y_0,\ x = 2\delta Q\\ |\hat{V}|^2 = |V|^2/|V_m|^2,\ |V_m|^2 = G_0/2G_v,\\ -\hat{G}_0 + j\hat{B}_0 = (-G_0 + jB_0)/Y_0\\ \hat{B}_\omega = B_\omega\omega_0/2QY_0 = 1\\ \hat{G}_v + j\hat{B}_v = (G_v + jB_v)|V_m|^2/Y_0 = \hat{G}_0/2 + j\hat{B}_v\\ \hat{P} = P/P_m,\ \hat{Y}_L = Y_L/Y_0 = \hat{G}_L + j\hat{B}_L\end{array}\right\} \tag{9}$$

Next, (7) is normalized as below.

$$\hat{B}_0 = (1 - \hat{G}_0)\hat{B}_v/\hat{G}_v = (1 - \hat{G}_0)2\hat{B}_v/\hat{G}_0 \tag{10}$$

Now, the normalized formula (8) can be determined.

2.2 Coefficients Determination Procedure

(1) To determine Y_0: the characteristic admittance of the line.
(2) To change $\hat{G}_L$ to find the maximum power P_m, when $\hat{B}_L = 0$. Let $\hat{G}_L$ at the power P_m be $\hat{G}_{Lpmax}$.

$$\hat{G}_0 = 2\hat{G}_{Lpmax},$$
$$\hat{G}_v = \hat{G}_0/2$$

(3) To measure the slope (K) of the constant frequency contour ($\Delta\omega/2\pi = 0$) that passes through the matched load ($\hat{G}_L = 1$, $\hat{B}_L = 0$) (Fig. 2).

$$\hat{B}_v = (\text{slope K}) \times \hat{G}_v = (\text{slope K}) \times \hat{G}_0/2$$
$$\hat{B}_0 = (1 - \hat{G}_0)\hat{B}_v/\hat{G}_v = (1 - \hat{G}_0)2\hat{B}_v/\hat{G}_0$$

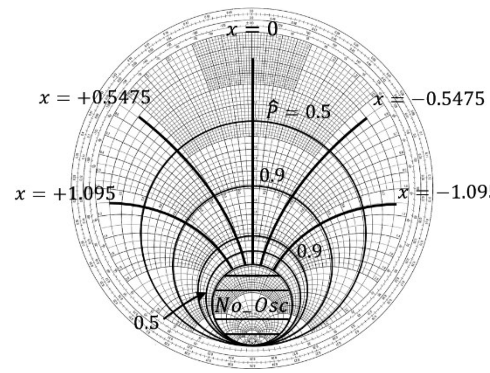

Fig. 3 Rieke diagram of van der Pol's model (Standardized diagram).

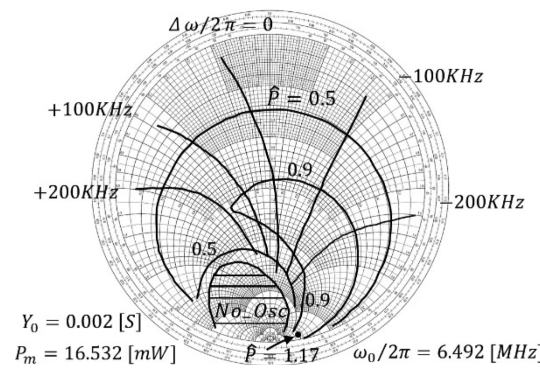

Fig. 4 Rieke diagram of High frequency Clapp oscillator.

Now, all coefficients of the normalized formula (8) are determined (Appendix).

2.3 Examples

A High Frequency (HF) Clapp oscillator is used to plot Rieke diagram using the circuit simulation, since the HF oscillator can easily change its circuit constant. The oscillator has a Field Effect Transistor (2SK241) and an inductor (9.4 μH). The connection capacitor 0.1μF between the oscillator and the load Y_L is selected to get the standardized Rieke diagram, which means that in the case of van der Pol's oscillator the center frequency line ($\Delta\omega/2\pi = 0$, $x = 0$) is equal to the zero susceptance and non-oscillation region exists in the heavy-load-admittance region [6] (Fig. 3).
(1) The characteristics of the line $Y_0 = 0.002$ [S].
(2) Pm = 16.532 [mW] and $\hat{G}_{Lpmax} = 1.388$, therefore

$$\hat{G}_0 = 2\hat{G}_{Lpmax} = 2.776, \quad \hat{G}_v = \hat{G}_0/2 = 1.388$$

(3) The slope (K) of the constant frequency contour ($\Delta\omega/2\pi = 0$) passing through the matched load ($\hat{G}_L = 1$, $\hat{B}_L = 0$). The slope (K) = +0.193, accordingly

$$\hat{B}_v = (\text{slope K}) \times \hat{G}_v = 0.268$$
$$\hat{B}_0 = (1 - \hat{G}_0)\hat{B}_v/\hat{G}_v = -0.343$$

As a results, the normalized formula of oscillator admittance is

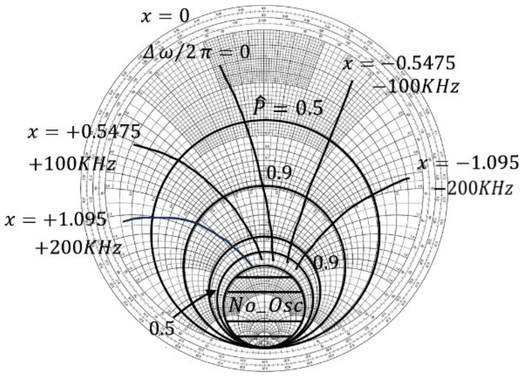

Fig. 5 Rieke diagram of normalized formula (11).

$$\hat{Y}(jx, |\hat{V}|^2) = -2.776 - j\,0.343 + jx + (1.388 + j\,0.268)|V|^2 \quad (11)$$

Since B_v is positive, the constant frequency contour ($\Delta\omega/2\pi = 0$) curves to the left toward the periphery (Fig. 5).

3. Conclusion

By limiting the measurement items to the slope (K) of the constant frequency contour ($\Delta\omega/2\pi = 0$) and the load conductance G_L which generates maximum output power P_m, the mathematical representation of microwave oscillator admittance can be determined by manual. The validity of the method was confirmed by comparison with the circuit simulation results (Fig. 4).

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Appendix: Method to Find B_v/G_v from B_ω

When the change in frequency and load susceptance are measured as 400 [KHz] and $2.19Y_0$ [S] respectively (Fig. 2), the coefficient B_ω is given by

$$B_\omega = 2.19 \times 0.002/(400 \times 2\pi)$$

$B_\omega = 0.1743 \times 10^{-5}$ [S/KHz]. Eliminating B_0 from (6) and (7). Considering $B_L = 0$,

$$(Y_0 - G_0)B_v/G_v = (G_L - G_0)B_v/G_v - B_\omega \Delta\omega \quad (\text{A}\cdot 1)$$

Taking Fig. A· 1 into consideration, B_v/G_v is given by

$$B_v/G_v = -B_\omega \Delta\omega_1/(Y_0 - G_L) \quad (\text{A}\cdot 2)$$

Substituting $G_L = 0.5Y_0$ [S], $\Delta\omega_1/2\pi = -17[KHz]$ and $Y_0 = 0.002$ [S] into (A· 2). Then $B_v/G_v = 0.186$.

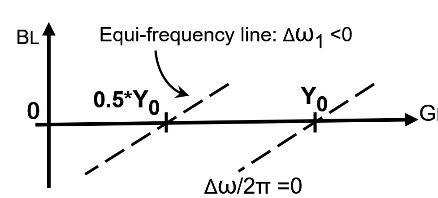

Fig. A· 1 Complex load plane (Slope: K > 0).