INVITED PAPER *Special Section on Recent Progress in Electromagnetic Theory and Its Application*

Reflection, Diffraction and Scattering at Low Grazing Angle of Incidence: Regular and Random Systems

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SUMMARY When a monochromatic electromagnetic plane wave is incident on an infinitely extending surface with the translation invariance property, a curious phenomenon often takes place at a low grazing angle of incidence, at which the total wave field vanishes and a dark shadow appears. This paper looks for physical and mathematical reasons why such a shadow occurs. Three cases are considered: wave reflection by a flat interface between two media, diffraction by a periodic surface, and scattering from a homogeneous random surface. Then, it is found that, when a translation invariant surface does not support guided waves (eigen functions) propagating with real propagation constants, such the shadow always takes place, because the primary excitation disappears at a low grazing angle of incidence. At the same time, a shadow form of solution is proposed. Further, several open problems are given for future works.
key words: *reflection, diffraction by periodic surface, scattering by random surface, shadow theory. reciprocity, guided wave*

1. Introduction

Wave reflection, diffraction and scattering at a low grazing angle of incidence (LGAI) are practically important in radar sensing of land and sea [1], [2]. When a monochromatic electromagnetic plane wave is incident on a surface with translation invariance property, however, a curious phenomenon often takes place at LGAI. The total wave field vanishes and physically becomes a dark shadow which we call the Fresnel shadow.

In the case of a flat interface, an exact solution indicates the reflected wave has the reflection coefficient equal to -1 and completely cancels the incident plane wave at LGAI [3]. In the case of a periodic grating, the 0th order diffraction amplitude (reflection coefficient) becomes -1 and any other order ones vanish at LGAI [4]–[6]. In the case of a randomly rough surface, approximate solutions by a probabilistic method [7]–[11] indicate that the incoherent scattering into all directions disappears and the coherent reflection coefficient becomes -1 at LGAI, which mean the Fresnel shadow.

Why does the Fresnel shadow take place at LGAI in these cases? What are conditions under which the Fresnel shadow appears? How can we explain the Fresnel shadow? This paper tries to answer these questions. Then, we find that, when a translation invariant surface does not support guided waves with real propagation constants, the Fresnel shadow takes place, because the primary excitation disappears at LGAI.

We only discuss the transverse magnetic (TM) case, where the time dependence $e^{-i\hat{\omega}t}$ with angular frequency $\hat{\omega}$ is assumed.

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2. Reflction and Transmission

Let us reconsider a well known problem: the reflection and transmission of a plane wave by an infinitely extended flat interface between two media (See Fig. 1.). Obviously, the interface is invariant under any translation in the x direction. We write the wave number k_m and impedance Z_m of the medium m as

$$k_m = \hat{\omega}\sqrt{\epsilon_m \mu_m}, \quad Z_m = \sqrt{\mu_m/\epsilon_m}, \quad (m = 0, 1). \tag{1}$$

Here, ϵ_m and μ_m are permittivity and permiability of the medium m. For simplicity, however, they are assumed to be real and to satisfy*

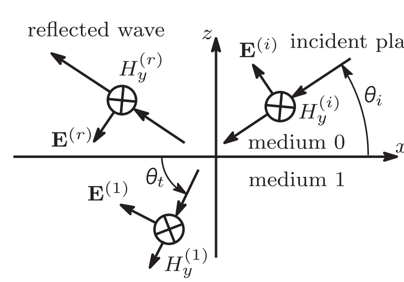

Fig. 1 Reflection and transmission of a TM plane wave by a flat interface ($z = 0$) between media 0 and 1.

*If the condition (4) is removed, guided waves (eigen functions) propagating along the interface could exist. If guided waves exist, (8) and (9) must be rewritten as

$$H_y^{(0)} = e^{-ipx - i\beta_0(p)z} + \Gamma(p)e^{-ipx + i\beta_0(p)z} + \sum_l A_l^{(0)}\psi_G^{(0)}(x, z|p_l), \tag{2}$$

$$H_y^{(1)} = T(p)e^{-ipx - i\beta_1(p)z} + \sum_l A_l^{(1)}\psi_G^{(1)}(x, z|p_l)., \tag{3}$$

Here, by $\psi_G^{(0)}(x, z|p_l)$ and $\psi_G^{(1)}(x, z|p_l)$ we denote a guided wave with propagation constant p_l, and $A_l^{(0)}$ and $A_l^{(1)}$ are constants. In our reflection problem, however, guided waves with complex propagation constants are suppressed physically, because the field must be finite even for $x \to \pm\infty$. Thus, the reflection problem has a unique solution and the Fresnel shadow takes place at LGAI, if and only if guided waves with real propagation constants do not exist.

$$\mu_0 = \mu_1, \quad k_1 > k_0 > 0. \tag{4}$$

By $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{e}_z$, we denote unit vectors into the x, y and z directions, respectively.

In the TM case, the electric field $\mathbf{E}^{(m)}$ is derived from $H_y^{(m)}$ the y component of the magnetic field as

$$\mathbf{E}^{(m)} = i(Z_m/k_m)\mathrm{rot}[H_y^{(m)}\mathbf{e}_y], \quad (m = 0, 1). \tag{5}$$

As is shown in Fig. 1, the total field $(\mathbf{E}^{(0)}, H_y^{(0)})$ in the medium 0 is a sum of the incident field $(\mathbf{E}^{(i)}, H_y^{(i)})$ and the reflected wave $(\mathbf{E}^{(r)}, H_y^{(r)})$, whereas $(\mathbf{E}^{(1)}, H_y^{(1)})$ represents the transmitted wave in the medium 1. The $H_y^{(m)}$ satisfies

$$[\partial^2/\partial x^2 + \partial^2/\partial z^2 + k_m^2]H_y^{(m)} = 0, \quad (m = 0, 1), \tag{6}$$

and the boundary conditions on the interface at $z = 0$,

$$H_y^{(0)} - H_y^{(1)} = 0, \quad \frac{1}{\epsilon_0}\frac{\partial H_y^{(0)}}{\partial z} - \frac{1}{\epsilon_1}\frac{\partial H_y^{(1)}}{\partial z} = 0, \quad (z = 0). \tag{7}$$

Next, we write the magnetic field as

$$H_y^{(0)} = e^{-ipx - i\beta_0(p)z} + \Gamma(p)e^{-ipx + i\beta_0(p)z}, \tag{8}$$
$$H_y^{(1)} = T(p)e^{-ipx - i\beta_1(p)z}, \tag{9}$$
$$p = k_0\cos(\theta_i). \tag{10}$$

Here, the first term in (8) represents the incident magnetic field $H_y^{(i)}$ and the second the reflected one $H_y^{(r)}$, where $\Gamma(p)$ is the reflection coefficient, $T(p)$ is the transmission coefficient and θ_i is the angle of incidence measured from the x axis (See Fig. 1). For real p, $\beta_m(p)$ is defined as

$$\beta_m(p) = \sqrt{k_m^2 - p^2},$$
$$Re[\beta_m(p)] \geq 0, \quad Im[\beta_m(p)] \geq 0, \quad (m = 0, 1), \tag{11}$$

where Re and Im represent real and imaginary part, respectively. From (10) and (11), we have at LGAI with $\theta_i \to 0$,

$$p \to k_0, \quad \beta_0(p) \to 0, \quad (\theta_i \to 0). \tag{12}$$

Note that the incident wave $e^{-ipx - i\beta_0(p)z}$ and the reflected wave $e^{-ipx + i\beta_0(p)z}$ are two independent solutions if $p \neq \pm k_0$ but degenerate at LGAI†.

As is well known, the boundary conditions (7) may be solved exactly and $\Gamma(p)$ and $T(p)$ are obtained as

$$\Gamma(p) + 1 = T(p) = \frac{2\beta_0(p)}{k_0}S(p), \tag{14}$$

$$S(p) = \frac{k_0/\epsilon_0}{\beta_0(p)/\epsilon_0 + \beta_1(p)/\epsilon_1}, \tag{15}$$

where $S(p)$ is the scattering factor. Due to the factor $\beta_0(p)/k_0 = \sin(\theta_i)$, the reflection coefficient $\Gamma(p)$ becomes -1 and the transmission coefficient $T(p)$ vanishes at LGAI (See Figs. 1. 22 and 1. 23, in Ref. [3] for examples.). As a result, the incident plane wave is completely cancelled by the reflected wave and the transmitted wave vanishes at LGAI [3]. Physically this means that the wave field becomes a dark shadow at LGAI, which we call the Fresnel shadow.

To represent the Fresnel shadow explicitly, we introduce the primary excitation $\psi_p(x, z)$ as a sum of the incident plane wave and a reflected wave with amplitude -1, and the elementary excitation $\psi_e(x, z)$ as,

$$\psi_p(x, z) = e^{-ipx}[e^{-i\beta_0(p)z} - e^{i\beta_0(p)z}] = \frac{2\beta_0(p)}{k_0}\psi_e(x, z), \tag{16}$$

$$\psi_e(x, z) = \frac{e^{-ipx}[e^{-i\beta_0(p)z} - e^{i\beta_0(p)z}]}{[2\beta_0(p)/k_0]} \tag{17}$$

where $\psi_p(x, z)$ is proportional to $\beta_0(p)$ and vanishes at LGAI. However, $\psi_e(x, z)$ becomes $-ik_0ze^{-ik_ox}$ at LGAI. Then, using (14), we rewrite (8) as

$$H_y^{(0)} = \psi_p(x, z) + [\Gamma(p) + 1]e^{-ipx + i\beta_0(p)z} \tag{18}$$
$$= \frac{2\beta_0(p)}{k_0}\mathcal{H}_y^{(0)}, \tag{19}$$
$$\mathcal{H}_y^{(0)} = \psi_e(x, z) + S(p)e^{-ipx + i\beta_0(p)z}, \tag{20}$$
$$H_y^{(1)} = \frac{2\beta_0(p)}{k_0}\mathcal{H}_y^{(1)}, \tag{21}$$
$$\mathcal{H}_y^{(1)} = S(p)e^{-ipx - i\beta_1(p)z}. \tag{22}$$

In (18) and (21), we regard $\psi_p(x, z)$ physically excites the modified reflected wave with the modified reflection coefficient $[\Gamma(p)+1]$ and the transmitted wave $H_y^{(1)}$. Since $\psi_p(x, z)$ is proportional to $\beta_0(p)$, $[\Gamma(p)+1]$ and $T(p)$ are proportional to $\beta_0(p)$, as is shown by (14). Separating the common factor $2\beta_0(p)/k_0$, we obtain shadow form solutions (19) and (21), which explicitly represent the Fresnel shadow at LGAI. In other words, the Fresnel shadow takes place, because the primary excitation vanishes at LGAI.

However, we call $\mathcal{H}_y^{(0)}$ and $\mathcal{H}_y^{(1)}$ the elementary fields, which are closely related to Green's function [20], [22]. We think of that the elementary fields should be first determined by (7), and then $\Gamma(p)$ and $T(p)$ should be calculated by (14). Using this idea, we will deal with the wave diffraction and scattering later.

2.1 Geometrical Explanation of the Fresnel Shadow

Let us consider geometrically why the Fresnel shadow appears at LGAI. Our solution is illustrated in Fig. 2.

†Let us write mathematical points first. When $p = k_0$, the magnetic filed in medium 0 is generally given, with aribitrary constants c_1 and c_2, by $H_y^{(0)} = (c_1 + c_2z)e^{-ik_ox}$. By use of $\psi_e(x, z)$ in (17), however, a general solution applicable for any real p is mathematically represented as

$$H_y^{(0)} = c_3\psi_e(x, z) + c_4e^{-ipx + i\beta_0(p)z}. \tag{13}$$

where c_3 and c_4 are constants independent of x and z. On the other hand, the incident wave component of $c_3\psi_e(x, z)$ must equal the first term in (8) for any real p. From this physical condition, we must set $c_3 = 2\beta_0(p)/k_0$. This means that (8) and (18) are complete expressions of the field even at LGAI in our reflection problem.

We start with an initial assumption such that electromagnetic fields exist in media 0 and 1 even at LGAI (See Fig. 2.). At LGAI, however, the incident and reflected plane waves degenerate into a transverse electromagnetic (TEM) wave propagating into the $-x$ direction and hence $\mathbf{E}^{(0)} = \mathbf{E}^{(i)} + \mathbf{E}^{(r)}$ has no x component, i.e., $\mathbf{E}^{(0)}\mathbf{e}_x = 0$. By Snell's law, however, the refraction angle θ_t in Fig. 2 becomes positive and less than $\pi/2$ under the condition (4). This means that the field in the medium 1 becomes a TEM wave propagating into the lower left direction and $\mathbf{E}^{(1)}$ has non-zero x component.

Next, let us apply the boundary condition on the interface at $z = 0$. Since the x component of the electric field is continuous across the interface, $\mathbf{E}^{(1)}\mathbf{e}_x = \mathbf{E}^{(0)}\mathbf{e}_x = 0$ holds for any x at $z = 0$. This means that $\mathbf{E}^{(1)} = 0$ holds identically in the medium 1, because $\pi/2 > \theta_t > 0$ and $\mathbf{E}^{(1)}$ is the electric field of a TEM wave. Since $\mathbf{E}^{(1)}$=0 in medium 1, the magnetic field $H_y^{(1)}$ vanishes identically. Since the magnetic field is also continuous across the interface, we obtain $H_y^{(0)} = H_y^{(1)} = 0$ for any x at $z = 0$. Since $H_y^{(0)}$ is the magnetic field of a TEM wave, $H_y^{(0)} = 0$ and $\mathbf{E}^{(0)} = 0$ hold identically in the medium 0. Thus, we conclude that Snell's law and the continuity of the electromagnetic field at the interface generate the Fresnel shadow at LGAI.

2.2 A Guided Wave on Perfectly Conductive Surface

The Fresnel shadow at LGAI occurs in general. When the medium 1 is perfectly conductive and (4) is unsatisfied, however, there exist guided waves with real propagating constants. The electric fields of guided waves are given as (See Fig. 3)

$$\mathbf{E}^{(0)} = A\mathbf{e}_z e^{\pm ik_0 x}, \tag{23}$$

where A is any number. Notice that (23) exactly satisfies Maxwell's equations and the boundary conation $\mathbf{e}_z \times \mathbf{E}^{(0)} = 0$ on the surface $z = 0$. Due to the existence of such a guided wave, $H_y^{(0)}$ cannot be determined in unique sense and the Fresnel shadow may not take place in the perfectly conductive flat case.

When the perfectly conductive surface becomes randomly rough, such a guided wave is expected to become a random leaky wave due to the surface scattering. However, no one obtains any solutions for such a random leaky wave yet [12].

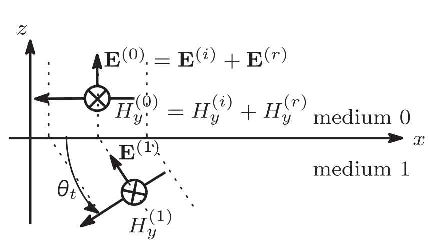

Fig. 2 Electromagnetic field at low grazing angle of incidence. A dotted line indicates an equi-phase plane.

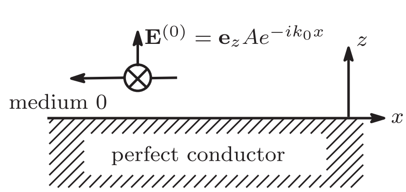

Fig. 3 A guided wave propagating along a flat surface.

3. Wave Diffraction by a Periodic Surface

Let us consider the diffraction of a TM plane wave by a perfectly conductive surface (See Fig. 4). We represent the periodic corrugation with the period L as

$$z = f(x) = f(x + L), \quad \sigma_h = \max[f(x)], \tag{24}$$

where σ_h denotes the highest excursion of the surface. The y component of the magnetic field $H_y^{(0)}$ satisfies (6) in the medium 0 and the Neumann condition

$$\partial H_y^{(0)}/\partial n|_{z=f} = 0 \tag{25}$$

on the surface (24), where $\partial/\partial n$ is normal derivative.

By (24), the surface corrugation is invariant under the translation by the period L, i.e. $f(x) \to f(x + L)$. By such invariance, there exists a solution $H_y^{(0)}(x, z)$ such that

$$H_y^{(0)}(x + L, z) = e^{-ipL} H_y^{(0)}(x, z), \tag{26}$$

which is a well known Floquet's theorem. In our opinion, there are several forms of $H_y^{(0)}(x, z)$ that satisfy (26). A form is given by

$$\text{(AM)} \quad H_y^{(0)} = e^{-ipx} A(x, z), \quad A(x + L, z) = A(x.z), \tag{27}$$

which we call the amplitude-modutaion (AM) representation. The Eq. (27) is widely used to represent the wave field in the region $z \geq \sigma_h$. However, we point out that the phase-amplitude modulation (PAM) representation

$$\text{(PAM)} \quad H_y^{(0)} = \exp\left(-i \int_0^x p'(\tau, z) d\tau\right) A'(x, z),$$
$$p'(x + L, z) = p'(x, z), \quad A'(x + L, z) = A'(x, z), \tag{28}$$

also satisfies (26), where $p'(x, z)$ and $A'(x, z)$ are periodic functions of x. The PAM representation is an analogy of the theory of waves in a homogeneous random media [13]. We believe that (28) is useful for the wave diffraction by a periodic surface when the period L is much larger than

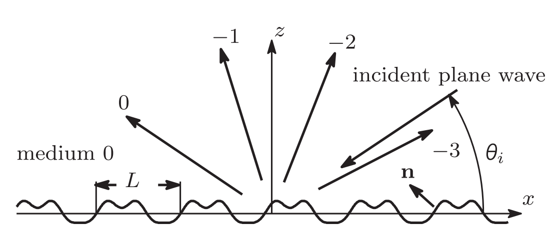

Fig. 4 Diffraction by a perfectly conductive periodic surface.

the wavelength. However, how to determine two periodic functions $p'(x,z)$ and $A'(x,z)$ is still open question.

3.1 Conventional Floquet's Form

From (27), the wave field is usually represented as

$$H_y^{(0)} = e^{-ipx-i\beta(p)z} + \sum_m A_m(p)e^{-i(p+mk_L)x+i\beta_0(p+mk_L)z}, \quad z \geq \sigma_h, \quad (29)$$

which is the conventional Floquet form. Here, k_L is the spatial angular frequency of the period L,

$$k_L = 2\pi/L. \quad (30)$$

The first term in (29) is the incident plane wave, p and $\beta_0(p)$ are defined by (10) and (11). The second term is a sum of up-going plane waves and evanescent waves, where $A_m(p)$ is the mth order diffraction amplitude and $A_0(p)$ is the reflection coefficient. However, note that (29) is valid when the diffraction problem is solved uniquely for any real p and when guided waves with real propagation constants do not exist.

3.2 Reciprocal Theorem

Diffraction amplitudes are determined approximately [4] or numerically [14]–[17]. At LGAI, however, many of them can be determined exactly by the reciprocity. Such determination was discussed in detail [18]. But we write only some important points here.

The reciprocal theorem [17] may be written as

$$\beta_0(p)A_m(-p-mk_L) = \beta_0(-p-mk_L)A_m(p), \quad (m = 0, \pm1, \pm2, \cdots), \quad (31)$$

which is exact and applicable for any periodic grating. Putting $m = 0$, we find $A_0(p) = A_0(-p)$, because $\beta_0(p) = \beta_0(-p)$ by (11). Putting $p = k_0$ and using $\beta_0(k_0) = 0$, one finds at LGAI for $m \neq 0$

$$\beta_0(-k_0 - mk_L)A_m(k_0) = 0, \quad (m = \pm1, \pm2, \cdots), \quad (32)$$

by which we will determine $A_m(k_0)$ below.

Single anomaly case. In the single anomaly case, $L \neq m\lambda/2$ holds for any positive integer m, $\lambda = 2\pi/k_0$ being wave length. Then, $\beta_0(-k_0 - mk_L) \neq 0$ holds for any integer $m(\neq 0)$. Thus, from (32) we obtain exactly

$$A_m(k_0) = 0, \quad (m = \pm1, \pm2, \pm3, \cdots), \quad (33)$$

by which (29) is reduced to a sum of two terms,

$$H_y^{(0)} = [e^{-ik_0x} + A_0(k_0)e^{-ik_0x}], \quad (34)$$

as is shown in Fig. 5. Since (29) is a sum of infinite terms, it could diverge for $z < \sigma_h$. However, (34) is free from such a divergence problem and can be applicable for any z. If the surface (24) is flat without any roughness, (34) becomes a guided wave. In a case of a corrugated surface, however, it can be shown [18] from (34) and (25)

$$A_0(k_0) = -1, \quad (35)$$

which means that the Fresnel shadow at LGAI always takes place in the single anomaly case.

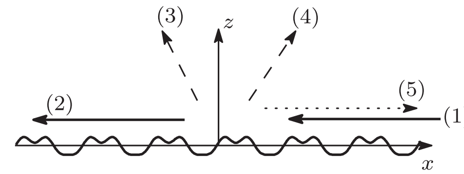

Fig. 5 Reciprocity. When a plane wave is incident with a low grazing angle from a direction (1), diffraction into directions (3) and (4) disappears. Only the reflected wave propagating into a grazing direction (2) may exist. Furthermore, a plane wave into another grazing direction (5) can exist if the periodic surface satisfies the double anomaly condition.

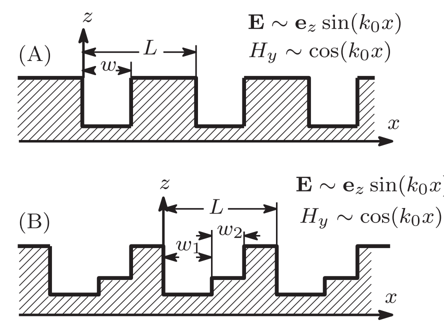

Fig. 6 Examples of perfectly conductive step gratings which support guided standing waves, when the period L and groove widths w, w_1, w_2 are integer multiples of $\lambda/2$, λ being wavelength.

Double anomaly case. In the double anomaly case where the period L is an integer multiple of $\lambda/2$, there exists an integer $\hat{m}(\neq 0)$ for which $\beta_0(-k_0 - \hat{m}k_L) = 0$ holds. Therefore, from (32) we obtain exactly

$$A_m(k_0) = 0, \quad m \neq 0, \hat{m}, \quad (36)$$

$$H_y^{(0)} = e^{-ik_0x} + A_0(k_0)e^{-ik_0x} + A_{\hat{m}}(k_0)e^{ik_0x}. \quad (37)$$

Since (37) must satisfy (25) on the periodic surface, we obtain in general

$$A_0(k_0) = -1, \quad A_{\hat{m}}(k_0) = 0, \quad (38)$$

which means $H_y^{(0)}$ vanishes for any x and z at LGAI.

However, an exception takes place in the double anomaly case [18]. For step gratings shown in Fig. 6, where L, w, w_1 and w_2 are integer multiples of $\lambda/2$, there exists a guided standing wave

$$H_y^{(0)} = Ae^{\pm ik_0x}[1 + e^{\mp 2ik_0x}] = 2A\cos(k_0x), \quad (39)$$

which may be understood as a guided wave with a real propagation constant k_0 or $-k_0$, and A is any constant. Note that (39) satisfies (6) and the boundary condition (25) on the surface of a step grating in Fig. 6.

In a previous paper [20], we started with a hypothesis such that the Fresnel shadow always takes place for any periodic gratings. However, the existence of guided standing waves means that such a hypothesis is imperfect and the uniqueness theorem does not hold for the TM wave case. Then, we conclude that the Fresnel shadow appears when and only when guided waves with real propagation constants do not exist.

3.3 Shadow Form of Solution

Assuming that such guided waves do not exist, we derive a shadow form of the diffracted field [20].

Using the primary excitation $\psi_p(x, z)$ in (16), we rewrite (29) as

$$H_y^{(0)} = \psi_p(x, z) + \sum_m [A_m(p) + \delta_{m0}] e^{-i(p+mk_L)x + i\beta(p+mk_L)z}. \quad (40)$$

Here, $[A_m(p)+\delta_{m0}]e^{-i(p+mk_L)x+i\beta(p+mk_L)z}$ is the mth order modified diffracted wave, which is excited by $\psi_p(x, z)$. Since $\psi_p(x, z)$ is proportional to $\beta_0(p)$, so is $[A_m(p) + \delta_{m0}]$. Thus, we may write

$$A_m(p) + \delta_{m0} = \frac{2\beta_0(p)}{k_0} S_m(p), \quad (41)$$

where $S_m(p)$ is the mth order scattering factor. By the reciprocity, we find

$$S_m(p - mk_L/2) = S_m(-p - mk_L/2). \quad (42)$$

Thus, $S_m(p)$ is symmetrical with respect to the symmetrical axis $p = -mk_L/2$, which is verified numerically [20]–[22].

Using (41), we obtain a shadow form of the diffracted field,

$$H_y^{(0)} = \frac{2\beta_0(p)}{k_0} \mathcal{H}_y^{(0)}, \quad (43)$$

$$\mathcal{H}_y^{(0)} = \psi_e(x, z) + \sum_m S_m(p) e^{-i(p+mk_L)x + i\beta_0(p+mk_L)z}. \quad (44)$$

Here, (43) explicitly represents that the Fresnel shadow takes place, because the primary excitation (16) is proportional to $\beta_0(p)$ and vanishes at LGAI.

In our opinion [20], however, scattering factors should be first determined from (44) and (25). Then, $A_m(p)$ should be calculated by (41).

3.4 Energy Conservation

In the grating theory, the diffraction efficiency is subject of interest. By use of the scattering factor, several properties of the diffraction efficiency become clear.

The energy conservation law may be written as

$$\sum_m \eta_m(p) = 1. \quad (45)$$

Here, $\eta_m(p)$ is the mth order diffraction efficiency which is given in terms of the scattering factor as [20],

(when $m \neq 0$)

$$\eta_m(p) = \begin{cases} \dfrac{4Re[\beta_0(p + mk_L)]\beta_0(p)|S_m(p)|^2}{k_0^2} & |p| \leq k_0 \\ \dfrac{4Re[\beta_0(p + mk_L)]|S_m(p)|^2}{k_0 Re[S_0(p)]} & |p| > k_0 \end{cases}$$

(when $m = 0$)

$$\eta_0(p) = \begin{cases} \left|1 - \dfrac{2\beta_0(p)}{k_0} S_0(p)\right|^2, & |p| \leq k_0 \\ 0, & |p| > k_0 \end{cases}. \quad (46)$$

These equations enable us to define $\eta_m(p)$ for a propagating wave incidence ($|p| < k_0$), an evanescent wave incidence ($|p| > k_0$) and even at LGAI ($p = \pm k_0$). By (46), we find $\eta_m(p)$ is discontinuous at $p = \pm k_0$, which is verified numerically [20]–[22]. When θ_i is real and goes to zero, we obtain

$$\lim_{\theta_i \to 0} \eta_m(k_0 \cos(\theta_i)) = \delta_{m0}, \quad (47)$$

which means that the 0 order diffraction efficiency becomes unity and any other order one vanishes at LGAI. This agrees with numerical results [14]–[16], [22].

4. Scattering from Randomly Rough Surface

Several analytical methods have been proposed for the scattering from a randomly rough surface [19]. However, it is quite difficult to exactly obtain an analytical solution. Numerical solutions are also difficult to obtain for an infinitely extending random surface.

However, we have proposed a probabilistic method which makes use of the translation invariance property of a homogeneous random function [13]. Assuming that the surface corrugation is mathematically given by a homogeneous random function, we have shown that the scattered wave has a stochastic Floquet's form, which is a product of an exponential phase factor and a homogeneous random function [7]. For a slightly rough case [8], [9], we have obtained an approximate solution, which indicates that the incoherent scattering into all directions disappears and only the coherent reflection occurs with the reflection coefficient −1 at LGAI. (See Figs. 3 and 7 in Ref. [9] for numerical examples.). By use of the reciprocity, this section newly demonstrates that such a curious phenomenon at LGAI takes place not only in a slightly rough case but also in general case.

Let us consider the scattering of a TM plane wave from a perfectly conductive random surface shown in Fig. 7. We assume the corrugation is given by a homogeneous Gaussian random function $f(x, \omega)$, where ω is a sample point in the sample space Ω. To express explicitly the translation invariance property, we represent $f(x, \omega)$ as [13],

$$z = f(x, \omega) = f(0, T^x \omega), \quad (48)$$

where T^x is a measure preserving transformation taking a

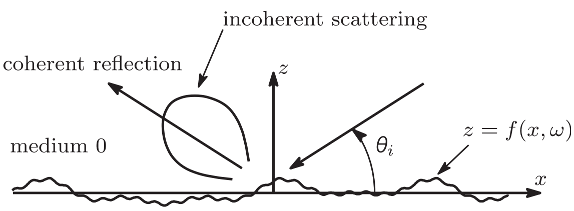

Fig. 7 Scattering from a homogeneous random surface. The coherent reflection into the specularly reflected direction and incoherent scattering into all directions.

sample point ω into another sample point $T^x\omega$. The right hand side indicates that $f(x, \omega)$ is invariant under a translation $(x, \omega) \to (x+a, T^{-a}\omega)$ for any a. Taking such translation invariance, we can find the scattered wave has a stochastic Floquet's form (51) below.

Let us represent the incident plane wave by $e^{-ipx-i\beta_0(p)z}$ and the scattered wave by $\psi_s(x, z, \omega)$. Then we write the total field as

$$H_y^{(0)} = e^{-ipx-i\beta_0(p)z} + \psi_s(x, z, \omega), \tag{49}$$

which satisfies (6) and the Neumann condition (25) on the surface (48), where p is given by (10).

Since the random surface is invariant under the translation above, the scattered wave must satisfy

$$\psi_s(x + a, z, T^{-a}\omega) = e^{-ipa}\psi_s(x, z, \omega), \tag{50}$$

which we call the stochastic Floquet theorem [7]. By use of a homogeneous random function $v(T^x\omega, z)$, a solution of (50) is given as

$$\psi_s(x, z, \omega) = e^{-ipx}v(T^x\omega, z), \tag{51}$$

which is a stochastic Floquet's form of solution. Note that, if we replace $v(T^x\omega, z)$ by a periodic function $A(x, z)$, (51) is reduced to the AM representation (27).

For concrete discussions, we write

$$z = f(0, T^x\omega) = \int_{-\infty}^{\infty} F(\lambda)e^{-i\lambda x}dB(\lambda, \omega),$$
$$F(\lambda) = F^*(-\lambda), \tag{52}$$

where the asterisk denotes complex conjugate, and $dB(\lambda, \omega)$ is a complex Gaussian random measure with

$$dB(\lambda, \omega) = dB^*(-\lambda, \omega), \tag{53}$$
$$\langle dB(\lambda, \omega)\rangle = 0, \tag{54}$$
$$\langle dB(\lambda, \omega)dB^*(\lambda', \omega)\rangle = \delta(\lambda - \lambda')d\lambda d\lambda' \tag{55}$$
$$dB(\lambda, T^{-a}\omega) = e^{i\lambda a}dB(\lambda, \omega). \tag{56}$$

Here, the angle brackets denote ensemble average over Ω. From these equations, we obtain

$$\langle f(x, \omega)\rangle = 0, \tag{57}$$
$$\sigma^2 = \langle f^2(x, \omega)\rangle = \int_{-\infty}^{\infty} |F(\lambda)|^2 d\lambda, \tag{58}$$

where σ is the root mean square surface height and $|F(\lambda)|^2$ is the power spectrum of the random surface. We assume $|F(\lambda)|^2$ is a continuous function of λ to make the Gaussian process $f(0, T^x\omega)$ ergodic.

Mathematically, $v(T^x\omega, z)$ is a functional of the random surface $f(0, T^x\omega)$. By (52), it is regarded as a stochastic functional of the complex Gaussian random measure $dB(\lambda, \omega)$ and is represented by the Wiener expansion [23] as

$$H_y^{(0)} = e^{-ipx-i\beta_0(p)z} + a_0(p)e^{-ipx+i\beta_0(p)z} + \int_{-\infty}^{\infty} a_1(\lambda|p)$$
$$\times e^{-i(p+\lambda)x+i\beta_0(p+\lambda)z}\hat{h}^{(1)}[dB(\lambda)] + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a_2(\lambda_1, \lambda_2|p)$$
$$e^{-i(p+\lambda_1+\lambda_2)x+i\beta_0(p+\lambda_1+\lambda_2)z}\hat{h}^{(2)}[dB(\lambda_1), dB(\lambda_2)]$$
$$+\cdots, \tag{59}$$

where we drop ω in $dB(\lambda, \omega)$ to simplify notations. $\hat{h}^{(1)}[dB(\lambda)]$, $\hat{h}^{(2)}[dB(\lambda_1), dB(\lambda_2)]$, $\cdots$ are random functions, called the Winener-Hermite differentials [23], with statistical properties:

$$\langle \hat{h}^{(n)}[dB(\lambda_1), dB(\lambda_2), \cdots, dB(\lambda_n)]\rangle = 0, \quad n \geq 1, \tag{60}$$
$$\langle \hat{h}^{(n)}[dB(\lambda_1), dB(\lambda_2), \cdots, dB(\lambda_n)]$$
$$\times\hat{h}^{(m)}[dB(\lambda_1), dB(\lambda_2), \cdots, dB(\lambda_m)]\rangle = 0, \quad m \neq n. \tag{61}$$

$a_0(p)$,$a_1(\lambda|p)$,$a_2(\lambda_1, \lambda_2|p)$, $\cdots$ are deterministic functions called Wiener kernels, which are implicitly assumed to be continuous with respect to their arguments. The expression (59) satisfies (6) term by term. Physically, integrals represent incoherent waves made up of up-going waves and evanescent waves with random amplitudes. Thus, the expression (59) is valid in a region above the highest excursion of the surface. In a random case, however, the highest excursion is difficult to define. However, we expect (59) is practically exact in the region $z >> \sigma$. Furthermore, (59) is a rigorous expression only when the scattered wave has a unique solution for any real p. This condition is implicitely assumed below.

From (59) and (60), we obtain the coherent wave field (average part) as

$$\langle H_y^{(0)}\rangle = e^{-ipx-i\beta_0(p)z} + a_0(p)e^{-ipx+i\beta_0(p)z}, \tag{62}$$

which is made up of the incident plane wave and the reflected wave with $a_0(p)$ the coherent reflection coefficient.

4.1 Reciprocity and Scattering Factor

Let us determine Wiener kernels at LGAI by the reciprocity. Reciprocity relations of Wiener kernels are given as [10],

$$\beta_0(p)a_n(\lambda_1, \lambda_2, \cdots, \lambda_n| - p - \lambda_1 - \lambda_2 - \cdots - \lambda_n)$$
$$= \beta_0(-p - \lambda_1 - \lambda_2 - \cdots - \lambda_n)a_n(\lambda_1, \lambda_2, \cdot\lambda, \lambda_n|p). \tag{63}$$

When $n = 0$, this means $a_0(p) = a_0(-p)$ because $\beta_0(p) = \beta_0(-p)$. Putting $p = k_0$ and using $\beta_0(k_0) = 0$, one finds, for

$n \geq 1$, $\beta_0(-k_0 - \lambda_1 - \lambda_2 - \cdots - \lambda_n)a_n(\lambda_1, \lambda_2, \cdot\lambda, \lambda_n|k_0) = 0$, which means

$$a_n(\lambda_1, \lambda_2, \cdots, \lambda_n|k_0) = 0, \quad n \geq 1, \tag{64}$$

since $a_n(\lambda_1, \lambda_2, \cdots, \lambda_n|k_0)$ is continuous with respect its arguments. Because of (64), all integrals in (59) vanish and the incoherent scattering disappears at LGAI. Thus, we have at LGAI.

$$H_y^{(0)} = e^{-ik_0x} + a_0(k_0)e^{-ik_0x}, \tag{65}$$

which is applicable for any z. Because (65) satisfies the Neumann condition on the random surface, we find the coherent reflection coefficient becomes −1 at LGAI,

$$a_0(k_0) = -1. \tag{66}$$

From this and (64), we may conclude that the Fresnel shadow always takes place at LGAI for a homogeneous Gaussian random surface, if guided waves with real propagation constants do not exist.

Next, let us obtain a shadow form of the scattered field. Using (16), we rewrite (59) as

$$\begin{aligned} H_y^{(0)} &= \psi_p(x,z) + [a_0(p)+1]e^{-ipx+i\beta_0(p)z} + \int_{-\infty}^{\infty} a_1(\lambda|p) \\ &\times e^{-i(p+\lambda)x+i\beta_0(p+\lambda)z}\hat{h}^{(1)}[dB(\lambda)] + \int\int_{-\infty}^{\infty} a_2(\lambda_1,\lambda_2|p) \\ &\times e^{-i(p+\lambda_1+\lambda_2)x+i\beta_0(p+\lambda_1+\lambda_2)z}\hat{h}^{(2)}[dB(\lambda_1), dB(\lambda_2)] \\ &+ \cdots\cdots. \end{aligned} \tag{67}$$

Here, the second term is the modified reflected wave. We regard again that $\psi_p(x,z)$ excites the modified reflected wave and incoherent waves. Since $\psi_p(x,z)$ is proportional to $\beta_0(p)$, the modified reflected wave and incoherent waves must be proportional to $\beta_0(p)$. Therefore, we may write

$$a_0(p) + 1 = \frac{2\beta_0(p)}{k_0}S_0(p), \tag{68}$$

$$a_n(\lambda_1, \lambda_2, \cdot\lambda, \lambda_n|p) = \frac{2\beta_0(p)}{k_0}S_n(\lambda_1.\lambda_2, \cdots, \lambda_n|p), \tag{69}$$

$$\begin{aligned} &S_n(\lambda_1.\lambda_2, \cdots, \lambda_n|p) \\ &\quad = S_n(\lambda_1.\lambda_2, \cdots, \lambda_n| - p - \lambda_1 - \lambda_2 - \cdots - \lambda_n), \end{aligned} \tag{70}$$

where $S_n(\lambda_1, \lambda_2, \cdots, \lambda_n|p)$ is the nth order scattering factor. The expressions (69) and (70) were first obtained in Ref. [10]. However, (68) is a new equation obtained in this paper.

Using these relations, we obtain a shadow form of the wave field as

$$H_y^{(0)} = \frac{2\beta_0(p)}{k_0}\mathcal{H}_y^{(0)}, \tag{71}$$

where the elementary field $\mathcal{H}_y^{(0)}$ is given by

$$\begin{aligned} \mathcal{H}_y^{(0)} &= \psi_e(x,z) + S_0(p)e^{-ipx+i\beta_0(p)z} + \int_{-\infty}^{\infty} S_1(\lambda|p) \\ &\times e^{-i(p+\lambda)x+i\beta_0(p+\lambda)z}\hat{h}^{(1)}[dB(\lambda)] + \int\int_{-\infty}^{\infty} S_2(\lambda_1,\lambda_2|p) \\ &\times e^{-i(p+\lambda_1+\lambda_2)x+i\beta_0(p+\lambda_1+\lambda_2)z}\hat{h}^{(2)}[dB(\lambda_1), dB(\lambda_2)] \\ &+ \cdots\cdots. \end{aligned} \tag{72}$$

Our shadow form (71) represents that the Fresnel shadow takes place because the primary excitation is proportional to $\beta_0(p)$ and vanishes at LGAI. We note that (71) and (72) give a rigorous expression in a Gaussian random surface case.

4.2 Approximate Solution

Let us obtain low order scattering factors in a slightly rough case with $k_0\sigma << 1$, First, we approximate the Neumann condition (25) to obtain an effective boundary condition on the $z = 0$ plane,

$$\left[-\frac{df}{dx}\frac{\partial}{\partial x} + \frac{\partial}{\partial z} + f(x,\omega)\frac{\partial^2}{\partial z^2}\right]H_y^{(0)} = 0, \quad (z = 0). \tag{73}$$

Substituting (71) and (72) into (73), we obtain a set of equations for scattering factors. Neglecting higher order scattering factors, we approximately obtain $S_0(p)$ and $S_1(\lambda|p)$ as

$$S_0(p) = \frac{k_0}{\beta_0(p) + Z_s(p)}, \tag{74}$$

$$S_1(\lambda|p) = -i\frac{k_0}{\beta_0(p) + Z_s(p)}\frac{[k_0^2 - p(p+\lambda)]F(\lambda)}{\beta_0(p+\lambda) + Z_s(p+\lambda)}, \tag{75}$$

$$Z_s(p) = \int_{-\infty}^{\infty}\frac{[k_0^2 - p(p+\lambda)]^2|F(\lambda)|^2}{\beta_0(p+\lambda) + Z_s(p+\lambda)}d\lambda, \tag{76}$$

where $Z_s(p)$ represents effects of multiple scattering [11]. Wiener kernels $a_0(p)$ and $a_1(\lambda|p)$ are obtained from (68), (69), (74) and (75). These kernels so obtained are essentially same as those in Ref. [9], where $Z_s(p+\lambda)$ in the integrand in (76) was neglected however. This example demonstrates that the elementary field (72) can be determined approximately at least for a slightly rough case. However, it is left for future work to determine scattering factors for a very rough case.

5. Conclusions

Wave reflection, diffraction and scattering of a plane wave by a translation invariance surface often becomes singular at LGAI. The total wave field vanishes and physically becomes a dark shadow which we call the Fresnel shadow. Such a curious phenomenon is discussed for three cases: reflection by a flat interface between two media, diffraction by a perfectly conductive periodic surface and scattering from a homogeneous Gaussian random surface. Then, we find that, when a translation invariant surface does not support guided waves with real propagation constants, the Fresnel shadow always takes place, because the primary excitation vanishes at LGAI. Also, we present a shadow form of solution. Further, we have presented several open questions to be solved.

Our discussions were restricted to a TM wave case, but can be applied to a transverse electric (TE) wave case. We note that the Fresnel shadow is expected to appear in another translation invariance cases, such as a periodic random surface [24], [25] and a homogeneous random slab [26]. However, the Fresnel shadow at LGAI may not appear in a case without translation invariance. A periodic grating with finite extent [27] is an example.

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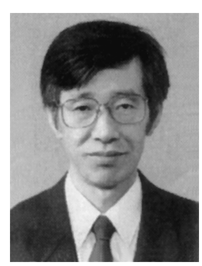

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