LETTER

Joint DOA and DOD Estimation Using KR-MUSIC for Overloaded Target in Bistatic MIMO Radars

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SUMMARY This letter deals with the joint direction of arrival and direction of departure estimation problem for overloaded target in bistatic multiple-input multiple-output radar system. In order to achieve the purpose of effective estimation, the presented Khatri-Rao (KR) MUSIC estimator with the ability to handle overloaded targets mainly combines the subspace characteristics of the target reflected wave signal and the KR product based on the array response. This letter also presents a computationally efficient KR noise subspace projection matrix estimation technique to reduce the computational load due to perform high-dimensional singular value decomposition. Finally, the effectiveness of the proposed method is verified by computer simulation.

key words: *bistatic radar, overload target, angle estimation, Khatri-Rao product, singular value decomposition*

1. Introduction

In recent years, the emerging multiple-input multiple-output (MIMO) radar technology has attracted immense interest in the radar industry [1]. In contrast to conventional bistatic radars, MIMO radars can implement angle estimation of the direction of arrival (DOA) and direction of departure (DOD) of multiple targets with asynchronous receiving and transmitting ends. Some adaptive technologies are applied in angle estimation of MIMO radar [2]–[7]. The two-dimensional (2D) minimum variance distortionless response (MVDR) [2] and multiple signal classification (MUSIC) [3] can automatically discern the 2D spatial angles problem at the receiver of the MIMO radar. In addition, if the targets are independently distributed, the uncorrelated reflection coefficients of the targets can be used to detect up to one less than the product of the numbers of receiving and transmitting antenna array elements. To reduce the computational load, the ESPRIT-root-MUSIC [4] technique used the ESPRIT and root-MUSIC approaches to estimate the DOA and DOD, respectively. Based on low-rank matrix reconstruction theory, a study [5] proposed a method different from the conventional vectorization method that used virtual sensor interpolation to process coprime arrays and obtain a uniform linear array (ULA) for generating covariance matrices. The scanning of bistatic radars can reach higher degrees of freedom compared with conventional radars, and these degrees of freedom can be used to improve the analytic abilities of angle estimation and overcome cluttering; however, if the number of targets to be detected exceeds that of the maximum analyzable targets (i.e., overloading occurs) [6], the estimation performance of the target direction and angle would largely decline, and the target would become undistinguishable. Thus, developing a method of implementing direction estimation with favorable analytic abilities in an environment with overloaded target numbers is a crucial and challenging research topic.

In this letter, a bistatic MIMO radar was used to process DOA and DOD estimation problems under the condition of overloaded target. If the transmitter and receiver arrays of a bistatic radar system are composed of ULA with M and N components, respectively, the vector dimension of the overall system output data is $MN \times 1$. The proposed method mainly uses the subspace characteristics of the correlation matrix of target reflection wave signals. The developed method involves the subspace formed by the Khatri-Rao (KR) product of array responses; thus, it is called the KR subspace approach. The KR subspace achieves meaningful results by enabling physically underdetermined DOA and DOD problems to be physically overdetermined under certain conditions. Although the degrees of freedom of the KR subspace is $(MN)^2$, a study [7] verified that the KR-MUSIC combining the KR subspace and MUSIC can clearly identify as many as $2MN - 2$ targets, which marks a notable improvement over the limit of $MN - 1$ targets of the MUSIC [3]. Moreover, to reduce the computational complexity, a computationally efficient approximate KR (AKR) noise subspace projection matrix estimation technique was presented for KR-MUSIC in this letter. Although it is similar to that the method which described in [8]. But the method in [8] is presented to reconstruct the conventional signal subspace and the noise subspace projection matrix by using eigenvalue decomposition for second order statistics. The KR-MUSIC uses full-dimensional correlation matrices data to construct subspaces, whereas the proposed AKR-MUSIC uses partial dimension vectorization correlation matrices as outputs and employed the Nyström method [9] to rebuild an AKR signal subspace and an AKR noise subspace projection matrix to reduce the computational load of high-dimensional singular value decomposition (SVD). Consequently, the simulation results can verify the effectiveness of the proposed method.

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2. Problem Formulation

2.1 Signal Model

A narrow-band bistatic MIMO radar system's transmitter and receiver arrays comprise ULA with M and N components, respectively. If all array components are omnidirectional antennas with unit amplitudes, the distances between transmitter and receiver array components can be represented as d_t and d_r, respectively. The transmitting antenna transmits orthogonal waveforms with identical bandwidths and center frequencies. In addition, K non-coherent targets were assumed to fall in the far field of the antenna arrays and in the range bin. The angle between the normal vector of the plane progressive wave of electromagnetic signals reflected by the target and the transmitter and receiver antennaarrays of the bistatic radar system was (θ_k, ϕ_k), where θ_k and ϕ_k are the DOA and DOD angles $(k = 1, 2, \cdots, K)$ of the kth target in relation to transmitter and receiver arrays. If K targets exist in an environment, the output passing the matched filter can be written as

$$\mathbf{y}(t) = \mathbf{G}(\theta, \phi)\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{G}(\theta, \phi) = [\mathbf{g}(\theta_1, \phi_1), \mathbf{g}(\theta_2, \phi_2), \cdots, \mathbf{g}(\theta_K, \phi_K)]$ is a steering matrix with dimensions $MN \times K$, $\mathbf{g}(\theta, \phi) = \mathbf{a}(\theta) \otimes \mathbf{b}(\phi)$, and $\otimes$ represents the Kronecker product. If Q is the number of transmitted pulse waves and $(\bullet)^T$ represents the transpose operation, the receiver antenna array with $N \times 1$ steering vector is $\mathbf{a}(\theta_k) = [1, e^{j2\pi d_r \sin\theta_k/\lambda}, e^{j2\pi 2d_r \sin\theta_k/\lambda}, \cdots, e^{j2\pi(N-1)d_r \sin\theta_k/\lambda}]^T$, and the transmitter array with $M \times 1$ steering vector is $\mathbf{b}(\phi_k) = [1, e^{j2\pi d_t \sin\phi_k/\lambda}, e^{j2\pi 2d_t \sin\phi_k/\lambda}, \cdots, e^{j2\pi(M-1)d_t \sin\phi_k/\lambda}]^T$, where λ is the wavelength. Because the receiver array receives the echo signals of the target, the echo signal vector is $\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_K(t)]^T$, where $s_k(t) = \beta_k e^{j\omega_{dk} t}$, β_k represents the sum of reflection coefficient and path loss determined by the radar cross-section of the kth target, and ω_{dk} represents the Doppler frequency corresponding to the kth target. $\mathbf{n}(t)$ represents the noise vector; its elements are assumed to have a Gaussian distribution with zero mean and variance σ_n^2.

2.2 KR-MUSIC Estimator

This subsection discusses using the KR-MUSIC [7] to perform DOA and DOD estimation for overloaded targets in bistatic radars. First, if the observation period of the echo signals is a steady-state process and the number of samples obtained through frame sampling is Q ($Q \geq MN$ and $E\{|s_k(t)|^2\} = \sigma_{fk}^2$ to satisfy statistical requirements), then the fth local correlation matrix $\mathbf{R}_f = E\{\mathbf{y}(t)\mathbf{y}^H(t)\} \in \mathbb{C}^{MN \times MN}$, $\forall t \in [(f-1)Q, fQ-1]$, where $f = 1, 2, \cdots, F$ and F is the number of frames. These local correlation matrices can be estimated according to the time average of the observation periods. In other words, $\hat{\mathbf{R}}_f = (1/Q)\sum_{t=(f-1)Q}^{fQ-1} \mathbf{y}(t)\mathbf{y}^H(t)$. Then, $\mathbf{R}_f$ is given by

$$\mathbf{R}_f = \mathbf{G}(\theta, \phi)\mathbf{D}_f\mathbf{G}^H(\theta, \phi) + \sigma_n^2\mathbf{I}_{MN} \tag{2}$$

where the correlation matrix of the reflection wave signal source of the fth frame $\mathbf{D}_f = diag\{\mathbf{d}_f\} \in \mathbb{R}^{K \times K}$, and $\mathbf{d}_f = [\sigma_{f1}^2, \sigma_{f2}^2, \cdots, \sigma_{fK}^2]^T$. If many local correlation matrices $\mathbf{R}_1, \mathbf{R}_2, \cdots, \mathbf{R}_F$ are available, the DOA and DOD $\{\theta_k, \phi_k\}$ of targets can be estimated based on $\{\mathbf{R}_1, \mathbf{R}_2, \cdots, \mathbf{R}_F\}$ without knowing the correlation matrices $(\mathbf{D}_1, \mathbf{D}_2, \cdots, \mathbf{D}_F)$ of local signal sources or the spatial noise covariance matrix $\sigma_n^2\mathbf{I}_{MN}$.

First, $\mathbf{R}_f$ is vectorized with $\mathbf{y}_f \triangleq \text{vec}\{\mathbf{R}_f\} = \mathbf{C}(\theta, \phi)\mathbf{d}_f + \text{vec}\{\sigma_n^2\mathbf{I}_{MN}\}$, where $\mathbf{C}(\theta, \phi) = \mathbf{G}^*(\theta, \phi) \odot \mathbf{G}(\theta, \phi) \in \mathbb{C}^{(MN)^2 \times K}$, vec$\{\bullet\}$ represents the vectorization of the matrix, and the symbol $\odot$ represents the KR product. Then, by stacking $[\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_F] \triangleq \mathbf{Y}$, the vectorized correlation matrix $\mathbf{Y}$ can be expressed as follows:

$$\mathbf{Y} = \mathbf{C}(\theta, \phi)\mathbf{\Psi}^T + \text{vec}\{\sigma_n^2\mathbf{I}_{MN}\}\mathbf{1}_F^T \tag{3}$$

where $\mathbf{C}(\theta, \phi) = [\mathbf{c}(\theta_1, \phi_1), \mathbf{c}(\theta_2, \phi_2), \cdots, \mathbf{c}(\theta_K, \phi_K)]$, $\mathbf{1}_F = [1, \cdots, 1]^T$, $\mathbf{\Psi} = [\mathbf{d}_1, \mathbf{d}_2, \cdots, \mathbf{d}_F]^T$, and $\mathbf{d}_f = [\sigma_{f1}^2, \sigma_{f2}^2, \cdots, \sigma_{fK}^2]^T$ with $f = 1, 2, \cdots, F$. $\mathbf{C}(\theta, \phi)$ is a virtual array response matrix, and $\mathbf{d}_f$ is a signal source vector. The virtual array dimension is $(MN)^2$. If $MN > 1$, the virtual array dimensions are greater than the actual array dimensions. If the number of array elements is lower than the number of targets, the degrees of freedom can be effectively increased to enable handling greater numbers of targets, namely identification signals in overloaded systems. Moreover, each row vector $\mathbf{d}_f$ on matrix $\mathbf{\Psi}$ describes the power of the reflection signal source relative to the frames or the variation of the long-term power distribution over time. In practice, the power distribution of the reflection wave signal source may differ in the time frame; thus, $\mathbf{\Psi}$ can maintain full rank.

The SVD of $\mathbf{Y}$ is defined as follows:

$$\mathbf{Y} = \begin{bmatrix} \mathbf{U}_s & \mathbf{U}_n \end{bmatrix} \begin{bmatrix} \Sigma_s & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_s^H \\ \mathbf{V}_n^H \end{bmatrix} \tag{4}$$

where $\mathbf{U}_s = [\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_K] \in \mathbb{C}^{(MN)^2 \times K}$, and $\mathbf{V}_s \in \mathbb{C}^{F \times K}$, respectively, represent the left and right singular matrices of nonzero singular values. $\mathbf{U}_n \in \mathbb{C}^{(MN)^2 \times (M^2N^2 - K)}$ and $\mathbf{V}_n \in \mathbb{C}^{F \times (M^2N^2 - K)}$ are left and right singular matrices, respectively, associated with these zero singular values. The diagonal of the diagonal matrix $\Sigma_s = diag\{[\sigma_1, \sigma_2, \cdots, \sigma_K]\} \in \mathbb{R}^{K \times K}$ contains nonzero singular values, where σ_k is singular value with $k = 1, 2, \cdots, K$. The target reflection wave signals satisfy $\mathbf{U}_n^H[\mathbf{G}^*(\theta_k, \phi_k) \odot \mathbf{G}(\theta_k, \phi_k) = \mathbf{U}_n^H[\mathbf{g}^*(\theta_k, \phi_k) \otimes \mathbf{g}(\theta_k, \phi_k)] = \mathbf{0}$ for $k = 1, 2, \cdots, K$. Thus, the KR subspace law for DOA and DOD estimation is expressed as follows:

$$\begin{aligned} &\text{Find } \{\theta, \phi\} \\ &\text{such that } \mathbf{U}_n^H\mathbf{c}(\theta, \phi) = \mathbf{0}, \{\theta, \phi\} \in [-90^\circ, 90^\circ] \end{aligned} \tag{5}$$

where $\mathbf{c}(\theta, \phi) = \mathbf{g}^*(\theta, \phi) \otimes \mathbf{g}(\theta, \phi)$. Similarly to the development of the subspace, if $\{\theta, \phi\}$ is the real DOA and DOD of the target, the inference in [7] suggests that (5) is satisfied and identifiable when the following two statements are true: the virtual array response matrix $\mathbf{G}^*(\theta,\phi)\odot\mathbf{G}(\theta,\phi)$ can generate full row rank (rank=K) if and only if $K \le 2MN-1$; the KR subspace law can be realized by using any real pair of angles in $\{\theta_k,\phi_k\}$, $k=1,2,\cdots,K$ if and only if $K \le 2MN-2$. In particular, under the KR subspace structure, undetermined DOA and DOD estimation can be conducted.

We can directly apply the KR subspace law of (5) to develop the KR-MUSIC for DOA and DOD estimation. Due to the orthogonality of the two subspaces, $\| \mathbf{U}_n^H\mathbf{c}(\theta_k,\phi_k) \|^2 = \mathbf{0}$, $k=1,2,\cdots,K$. The conventional approach to estimate DOA and DOD concurrently involves the direction-finding approach involves searching the peak value in the virtual spectrum. The searching function is given by

$$P_{\text{KR-M}}(\hat{\theta},\hat{\phi}) = \text{Max}[\mathbf{c}^H(\theta,\phi)\mathbf{U}_n\mathbf{U}_n^H\mathbf{c}(\theta,\phi)]^{-1} \quad (6)$$

where $\mathbf{U}_n$ is the KR noise subspace derived from (4). $\mathbf{c}(\theta,\phi) = \mathbf{g}^*(\theta,\phi)\otimes\mathbf{g}(\theta,\phi)$ is the spatial-domain direction scanning vector, with the direction angles corresponding to the peak values being the estimated DOA and DOD.

The KR-MUSIC is expected to enable DOA and DOD estimation on $K \le 2MN-2$ targets. Given the received signal $\{\mathbf{y}(t)\}$, target number K, frame number F, and the number of samples Q obtained in each frame, the KR-MUSIC is implemented as follows:

Step 1. Calculate $\{\hat{\mathbf{R}}_f\}_{f=1}^F$.

Step 2. Form $\mathbf{Y} = [\text{vec}\{\hat{\mathbf{R}}_1\}, \text{vec}\{\hat{\mathbf{R}}_2\},\cdots,\text{vec}\{\hat{\mathbf{R}}_F\}]$.

Step 3. Extract KR subspaces: Conduct SVD on $\mathbf{Y}$ and obtain $\mathbf{U}_n$ from $\mathbf{U}$.

Step 4. KR-MUSIC: Calculate spectrum $P_{\text{KR-M}}(\theta,\phi)$ on $\{\theta,\phi\} \in [-90°, 90°]$ and identify angles corresponding to the K peak values of $P_{\text{KR-M}}(\theta,\phi)$, which are the results of DOA and DOD estimation.

Although this KR subspace technique can handle overloaded systems, the computational load will increase substantially if the matrix dimensions is augmented from MN to $(MN)^2$, particularly when extracting SVD subspace.

3. AKR-MUSIC Estimator

The AKR-MUSIC estimator exploits the Nyström version to reconstruct the KR noise subspace, and its key point is using a suitable dimensions N_a to achieve the trade-off between computational complexity and estimation performance. First, if the dimensions of the vectorized correlation matrix $\mathbf{Y}$ are $(MN)^2\times F$, data matrix $\tilde{\mathbf{Y}}$ with $N_\text{a}\times F$ dimensions represents the output of extracting elements from row 1 to row N_a of $\mathbf{Y}$. Then, the SVD of $\tilde{\mathbf{Y}}$ can be calculated as $\tilde{\mathbf{Y}} = \tilde{\mathbf{U}}_{\tilde{Y}}\tilde{\Sigma}_{\tilde{Y}}\tilde{\mathbf{V}}_{\tilde{Y}}^H$, where $\tilde{\mathbf{U}}_{\tilde{Y}} = [\tilde{\mathbf{U}}_s,\tilde{\mathbf{U}}_n] = [\tilde{\mathbf{u}}_1,\tilde{\mathbf{u}}_2,\cdots,\tilde{\mathbf{u}}_{N_\text{a}}]$ is the matrix formed from the left singular vector with dimensions $N_\text{a}\times N_\text{a}$, $\tilde{\mathbf{V}}_{\tilde{Y}}$ is the matrix formed from the right singular vector with dimensions $F\times F$, $\tilde{\Sigma}_{\tilde{Y}} \in \mathbb{R}^{N_\text{a}\times F}$ is the singular value matrix with singular values $\tilde{\sigma}_i$, and square matrices $\tilde{\mathbf{U}}_{\tilde{Y}}$ and $\tilde{\mathbf{V}}_{\tilde{Y}}$ are unitary matrices. If $N_\text{a} > K$, the subspace $\tilde{\mathbf{U}}_s$ with is extended from left singular matrix $\tilde{\mathbf{U}}_{\tilde{Y}}$ is equivalent to $\tilde{\mathbf{C}}(\theta,\phi)$, which represents the extraction of elements from row 1 to row N_a of $\mathbf{C}(\theta,\phi)$. Then, let $\mathbf{R}_{\tilde{Y}\tilde{Y}} = \tilde{\mathbf{Y}}\tilde{\mathbf{Y}}^H$ and satisfy $\mathbf{R}_{\tilde{Y}\tilde{Y}}\tilde{\mathbf{u}}_i = \tilde{\sigma}_i^2\tilde{\mathbf{u}}_i$, $i=1,2,\cdots,N_\text{a}$. To realize the Nyström approximation, the correlation matrix $\mathbf{R}_{Y\tilde{Y}} = \mathbf{Y}\tilde{\mathbf{Y}}^H$ with dimensions $(MN)^2\times N_\text{a}$ is given by

$$\mathbf{R}_{Y\tilde{Y}} = \mathbf{C}(\theta,\phi)\mathbf{D}\tilde{\mathbf{C}}^H(\theta,\phi) + \sigma_n^2\mathbf{I}_{(MN)^2\times N_\text{a}} \quad (7)$$

where $\mathbf{I}_{(MN)^2\times N_\text{a}}$ is a matrix with $(MN)^2\times N_\text{a}$ dimensions and diagonal elements of 1; the remaining elements are 0. The principal left singular vector of $\mathbf{Y}$, $\{\mathbf{u}_1,\mathbf{u}_2,\cdots,\mathbf{u}_K\}$, has an approximate vector $\{\widehat{\mathbf{u}}_1,\widehat{\mathbf{u}}_2,\cdots,\widehat{\mathbf{u}}_K\}$ that satisfies $\mathbf{R}_{Y\tilde{Y}}\tilde{\mathbf{u}}_i = \tilde{\sigma}_i^2\widehat{\mathbf{u}}_i$. The approximate vector of $\{\mathbf{u}_1,\mathbf{u}_2,\cdots,\mathbf{u}_K\}$ can be further expressed as $\widehat{\mathbf{u}}_i = (1/\tilde{\sigma}_i^2)\mathbf{R}_{Y\tilde{Y}}\tilde{\mathbf{u}}_i$. If $\{\widehat{\mathbf{u}}_1,\widehat{\mathbf{u}}_2,\cdots,\widehat{\mathbf{u}}_K\}$ represents the K approximate principal singular vectors (eigenvectors) of $\mathbf{R}_{YY}$, then the AKR signal subspace (defined as $\widehat{\mathbf{U}}_s$) can be expressed as $\widehat{\mathbf{U}}_s = [\widehat{\mathbf{u}}_1,\widehat{\mathbf{u}}_2,\cdots,\widehat{\mathbf{u}}_K]$. Because of the orthogonality of singular vectors, $\widehat{\mathbf{U}}_s\widehat{\mathbf{U}}_s^H + \widehat{\mathbf{U}}_n\widehat{\mathbf{U}}_n^H = \mathbf{I}_{(MN)^2}$. According to unitary theorem [3], the orthogonal projection matrix of the AKR noise subspaces can be expressed as $\widehat{\mathbf{U}}_n\widehat{\mathbf{U}}_n^H = \mathbf{I}_{(MN)^2} - \widehat{\mathbf{U}}_s\widehat{\mathbf{U}}_s^H$. The projection matrix of the AKR noise subspace is extracted using the following steps:

Step 1. Obtain $\mathbf{Y}$ and select an appropriate N_a that satisfies $N_\text{a} > K$ to construct $\tilde{\mathbf{Y}}$.

Step 2. Determine $\{\tilde{\mathbf{u}}_i,\tilde{\sigma}_i\}_{i=1}^{N_\text{a}}$ by conducting SVD on $\tilde{\mathbf{Y}}$.

Step 3. Calculate $\mathbf{R}_{Y\tilde{Y}} = \mathbf{Y}\tilde{\mathbf{Y}}^H$.

Step 4. Calculate $\widehat{\mathbf{u}}_i = (1/\tilde{\sigma}_i^2)\mathbf{R}_{Y\tilde{Y}}\tilde{\mathbf{u}}_i$ and use the singular vectors corresponding to the first K largest singular values to construct $\widehat{\mathbf{U}}_s = [\widehat{\mathbf{u}}_1,\widehat{\mathbf{u}}_2,\cdots,\widehat{\mathbf{u}}_K]$.

Step 5. Calculate $\widehat{\mathbf{U}}_n\widehat{\mathbf{U}}_n^H = \mathbf{I}_{(MN)^2} - \widehat{\mathbf{U}}_s\widehat{\mathbf{U}}_s^H$.

Finally, Step 3 of the KR-MUSIC implementation steps is replaced with the steps of extracting the projection of AKR noise subspaces, and the function used in Step 4 of the KR-MUSIC is replaced with the following spectral searching function:

$$P_{\text{AKR-M}}(\hat{\theta},\hat{\phi}) = \text{Max}[\mathbf{c}^H(\theta,\phi)\widehat{\mathbf{U}}_n\widehat{\mathbf{U}}_n^H\mathbf{c}(\theta,\phi)]^{-1} \quad (8)$$

Then, the obtained AKR-MUSIC can be used to efficiently estimate DOA and DOD.

4. Computational Complexity Analysis

In this section, the number of complex multiplications (CM) of the KR-MUSIC and AKR-MUSIC are evaluated. Assuming K targets, N receiving antennas, M transmitting antennas, and F frames. For each test, the computational complexities of calculating SVD for an $(MN)^2\times F$ correlation matrix requires $12(MN)^6$ CM [10]. In addition, the computational complexities of calculating $\{\hat{\mathbf{R}}_f\}_{f=1}^F$ for an $MN\times Q$ matrix require $F[(MN)^2Q]$ CM. Let F_θ and F_ϕ be the number of searches for θ and ϕ, respectively. The searching function of the KR-MUSIC and AKR-MUSIC are (6) and (8), respectively. Hence, the required CM of the KR-MUSIC and AKR-MUSIC are $F_\theta F_\phi\{2(MN)^2[(MN)^2-K]+(MN)^2\}$ and

Table 1 Computational complexity analysis.

Estimators	Computational complexities
KR-MUSIC	$12(MN)^6+F[(MN)^2Q]+F_\theta F_\phi\{2((MN)^2[(MN)^2-K]+(MN)^2\}$
AKR-MUSIC	$12N_\mathrm{a}^3+F[(MN)^2Q]+(MN)^2FN_\mathrm{a}+K[2(MN)^2N_\mathrm{a}]+(MN)^4K+F_\theta F_\phi\{(MN)^4+(MN)^2\}$

$F_\theta F_\phi\{(MN)^4+(MN)^2\}$, respectively. In particular, the AKR-MUSIC calculates the SVD of the matrix $\tilde{\mathbf{Y}}$ of the $N_\mathrm{a}\times F$ dimension and requires $12N_\mathrm{a}^3$ CM. The matrix $\mathbf{R}_{Y\tilde{Y}}=\mathbf{Y}\tilde{\mathbf{Y}}^H$ and the AKR signal subspace $\widehat{\mathbf{U}}_s=[\widehat{\mathbf{u}}_1,\widehat{\mathbf{u}}_2,\cdots,\widehat{\mathbf{u}}_K]$ are computed, which require $(MN)^2FN_\mathrm{a}$ and $K[2(MN)^2N_\mathrm{a}]$ CM, respectively. Calculating the projection matrix $\widehat{\mathbf{U}}_n\widehat{\mathbf{U}}_n^H=\mathbf{I}_{(MN)^2}-\widehat{\mathbf{U}}_s\widehat{\mathbf{U}}_s^H$ of the AKR noise subspace requires $(MN)^4K$ CM. Briefly, the required CM are listed in Table 1.

5. Simulation Results

The results of computer simulations were used to compare the performance of the proposed AKR-MUSIC, KR-MUSIC, MVDR [2], MUSIC [3], and ESPRIT-root-MUSIC [4] estimators. For all simulations, the number of uncorrelated aerial targets is $K=9$, their DOA and DOD in the bistatic radars are $(\theta_1,\phi_1)=(-20°,-55°)$, $(\theta_2,\phi_2)=(-10°,35°)$, $(\theta_3,\phi_3)=(0°,-25°)$, $(\theta_4,\phi_4)=(10°,15°)$, $(\theta_5,\phi_5)=(20°,-5°)$, $(\theta_6,\phi_6)=(30°,45°)$, $(\theta_7,\phi_7)=(40°,-5°)$, $(\theta_8,\phi_8)=(50°,25°)$, $(\theta_9,\phi_9)=(60°,5°)$. Unless stated otherwise, the DOA and DOD estimation for non-overloaded targets yields in $M=5$ and $N=3$. Notably, the scenario satisfies the non-overloaded target condition of $(MN-1)>K$. For overloaded targets, the DOA and DOD estimation yields in $M=3$ and $N=3$, which satisfies the overloaded target condition of $(MN-1)<K$. The snapshot number of each frame in the KR-MUSIC is fixed at $Q=600$, and the frame number is $F=10^2$. The signal-to-noise ratio (SNR) of target reflection signal is $\mathrm{SNR}=10\log_{10}E[s_k(t)^2]/\sigma_n^2$. The MVDR and MUSIC spectral searching grid angle was set $\mu=0.1°$ and the KR-MUSIC and AKR-MUSIC was set to $\mu=1°$. The interval between the antenna components is half-wavelength, and all array elements are assumed to have omnidirectional unit gain. The total root mean square error (TRMSE) of the DOA and DOD estimation of K targets, $\mathrm{TRMSE}=\sum_{k=1}^{K}[(\hat{\theta}_k-\theta_k)^2+(\hat{\phi}_k-\phi_k)^2]^{0.5}$, was adopted as the performance indicator, and 10^3 Monte Carlo tests were performed with various parameter settings.

Figure 1 depicts the performance of the AKR-MUSIC by revealing the dimension numbers N_a of the output received data arrays with non-overloaded and overloaded targets at SNR = 15 dB. To support the performance evaluation, the simulation results use the KR-MUSIC performance as a reference for comparison. Because approximation was used, the number of dimensions changed from $(MN)^2\times(MN)^2$ to $(MN)^2\times N_\mathrm{a}$, it clearly indicates that AKR-MUSIC has a TRMSE of 5×10^{-3} in $N_\mathrm{a}\geq 10$ with both non-overloaded and overloaded targets. The performance of the AKR-MUSIC is roughly comparable to that of the KR-MUSIC. Thus, in the

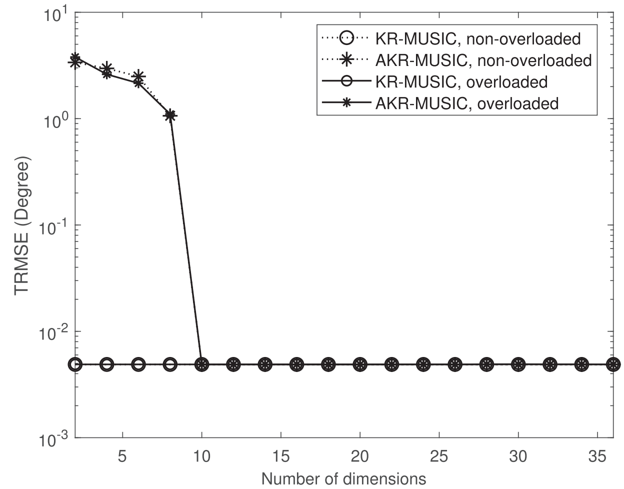

Fig. 1 TRMSE versus number of dimensions.

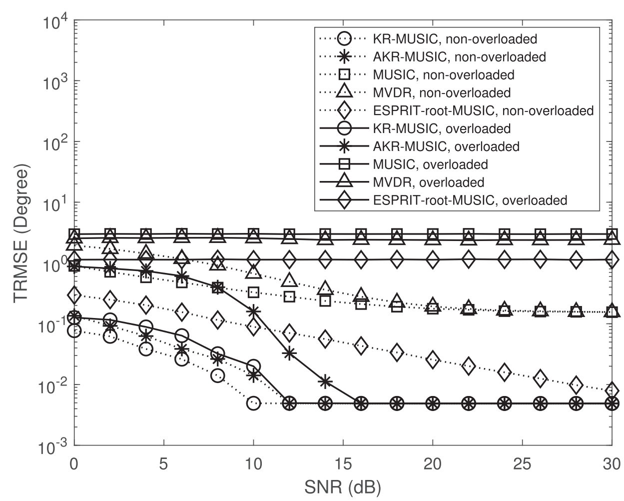

Fig. 2 TRMSE versus SNR.

following simulation process, $N_\mathrm{a}=12$ was selected for the AKR-MUSIC to minimize the computational load. Figure 2 presents the comparison of angle estimation performance of estimators under different SNRs. With non-overloaded targets, MUSIC, in which an orthogonal relationship exists between the noise subspace and the incidence steering vector, had superior estimation performance to the MVDR when SNR was low. The ESPRIT-root-MUSIC achieves favorable estimation performance due to its root-finding approach, which has no resolution limit. However, this figure reveals that if SNR $\geq$ 10 dB, the performance of the AKR-MUSIC and KR-MUSIC are similar. With overloaded targets, conventional methods have poor estimation performance because they can handle only $MN-1$ targets. By contrast, the KR-MUSIC and AKR-MUSIC normally despite overloading. Figure 3 presents the TRMSE of DOA and DOD estimation with various snapshot numbers at SNR = 15 dB to reveal the estimator convergence. The proposed AKR-MUSIC adopts subspace approximation, which leads to slower convergence compared with the KR-MUSIC. It not only presents

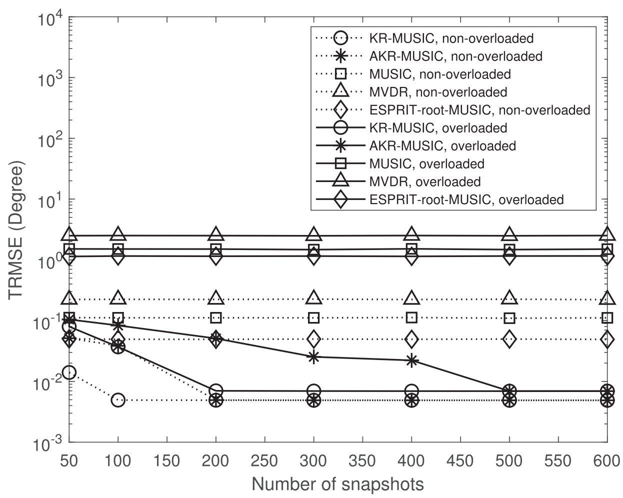

Fig. 3 TRMSE versus number of snapshots.

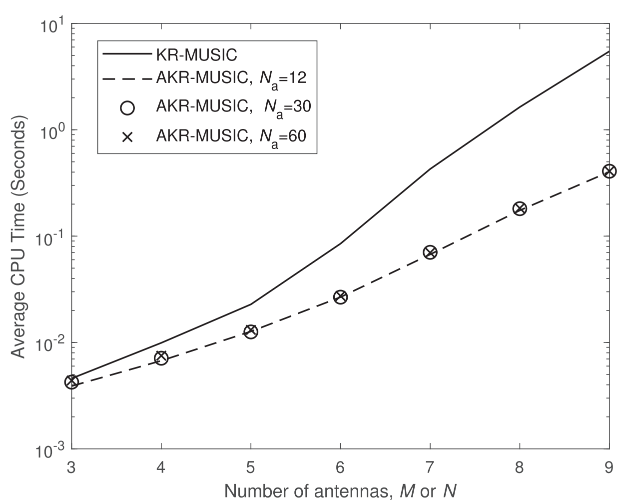

Fig. 4 Computational time versus number of antennas.

improved computational performance, but also achieves favorable angle estimation performance. Figure 4 presents an evaluation of the computational complexity using the TIC and TOC instruction in MATLAB. For each Monte-Carlo trail, the calculation of CPU time consuming (in seconds) of each estimator started at the computation of $\{\hat{\mathbf{R}}_f\}_{f=1}^{F}$ and terminated at the outputting noise subspace or orthogonal projection matrix. Assume that the number of transmit antennas M is the same as the number of receive antennas N. The average CPU time is plotted versus number of antennas (M or N) with $N_a = \{12, 30, 60\}$. As the result of Fig. 4, we can observe that the average CPU time of all AKR-MUSIC are less than the average CPU time for the KR-MUSIC, particularly when antenna becomes larger. Again, this figure is presented to verify the efficiency of the AKR-MUSIC.

6. Conclusion

A joint DOA and DOD estimation method based on the KR subspaces for bistatic MIMO radars was proposed in this letter. The method involves implementing SVD on partial-dimensionally vectorized correlation matrices. The KR-MUSIC is superior to the conventional estimators for processing overloaded targets. However, it involves a considerably increased computational load due to the increase of matrix dimensions from MN to $(MN)^2$. To reduce the computational load while maintaining the advantages in processing overloaded targets, the AKR-MUSIC increases analytic abilities and increases the computational efficiency for overloaded targets. Simulation results verified the effectiveness of the proposed AKR-MUSIC.

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