LETTER

A Hypergraph Matching Labeled Multi-Bernoulli Filter for Group Targets Tracking

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SUMMARY This paper addresses the association problem of tracking closely spaced targets in group or formation. In the Labeled Multi-Bernoulli Filter (LMB), the weight of a hypothesis is directly affected by the distance between prediction and measurement. This may generate false associations when dealing with the closely spaced multiple targets. Thus we consider utilizing structure information among the group or formation. Since, the relative position relation of the targets in group or formation varies slightly within a short time, the targets are considered as nodes of a topological structure. Then the position relation among the targets is modeled as a hypergraph. The hypergraph matching method is used to resolve the association matrix. At last, with the structure prior information introduced, the new joint cost matrix is re-derived to generate hypotheses, and the filtering recursion is implemented in a Gaussian mixture way. The simulation results show that the proposed algorithm can effectively deal with group targets and is superior to the LMB filter in tracking precision and accuracy.

key words: *group targets structure information, hypergraph matching, joint cost matrix, labeled multi-Bernoulli*

1. Introduction

The group targets are a set of multiple targets with stable relative position relationship and move coordinately in the surveillance region. Since the targets in group are closely spaced, the main difficulty in achieving the trajectories of the group targets is the data association between the predictions and the measurements in the context of process noise and measurement noise.

The association logic of conventional multi-target tracking algorithms such as joint probability data association (JPDA) [1] and multi-hypothesis tracking (MHT) [2], [3] are not reliable when dealing with densely distributed targets. As Mahler and Vo constantly improved the random finite set (RFS) theory, new ways are presented for multi-target tracking, forming the following three new solutions:

- **PHD combined with MHT**. Kusha Panta proposed two methods of combining probability hypothesis density (PHD) with MHT. One is to use the output of the PHD filter as the input of MHT (PHD-with-association filter), which is equivalent to the cascade of the two algorithms [4]. Another is regarding the PHD filter as a clutter filter, and the measurements around the output of the PHD filter is used as the input of the MHT (PHD-with-MHT) [5]. These methods are superposition of the two algorithms. It increases the complexity of the algorithm, and the performance is determined by only one of the two algorithms.
- **PHD with label attached**. By attaching a unique label to the Gaussian component [6] or particle [7], the estimation state of each target carries a unique identity. In this way, the target ID is embedded in the PHD recursion process with no algorithm complexity increases. However, when dealing with closely spaced targets, the results of the clustering is unreliable, and it is hard to extract the state and target label.
- **GLMB and LMB**. The latest proposed Generalized Labeled Multi-Bernoulli (GLMB) [10], [11] combines MHT and RFS statistical theory and has been shown to outperform PHD, cardinalized PHD (CPHD), and cardinality balanced multi-target multi-Bernoulli (CBMeMBer) filters in tracking accuracy. It inherits the good performance of MHT algorithm in the context of low detection probability and dense clutter. The LMB [12] filter, i.e., the simplified version of GLMB, solved the combination explosion problem and has been applied successfully in large scale multi-target tracking.

To further improve the tracking performance of closely spaced targets, the prior information among the group should not be ignored. Although the group targets are densely distributed, the relative position is stable within a short time, and it is an effective information that can be used. Hypergraph is a generalization of a graph and it is capable of representing relationship among vertexes. Hypergraph matching method was always used in feature point matching in computer vision, pattern recognition, and machine learning fields. Recently, the hypergraph was introduced in point target tracking. In [8], the authors described the group structure under the framework of hypergraph theory and improved the performance of the data association. In this paper, we introduce the group structure information into the LMB filter, expecting to improve the data association performance and the ability of the group target tracking.

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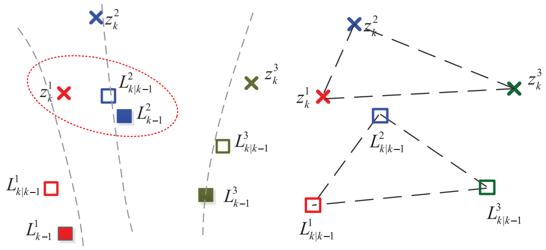
2. Association Problem in Group Targets Tracking

2.1 Disadvantage of Association Based on Distance

In LMB filter, the association probability is determined by



(a) association based on distance (b) association based on group structure

Fig. 1 Comparison of the two association logics. The solid squares denote the tracks at time step $k-1$, and the hollow ones are the predictions. $\times$ are measurements at time step k. Different colors represent different IDs.

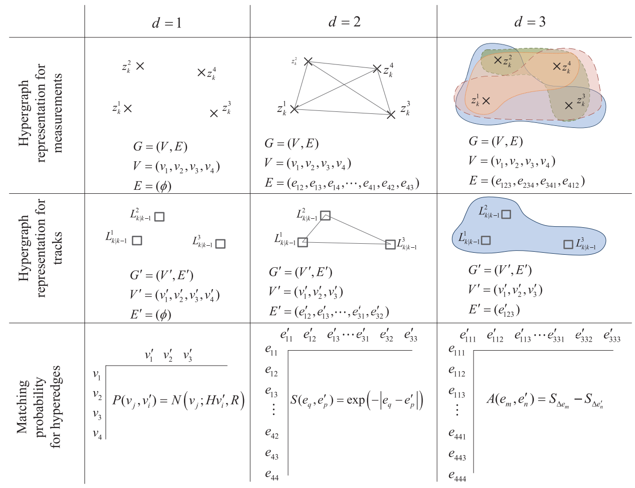


Fig. 2 The hypergraph representations of tracks and measurements.

the distance between prediction and measurement, but this strategy is no longer reliable when tracking closely spaced multi-targets. As shown in Fig. 1 (a), compared with track $L^1_{k|k-1}$, the measurement z^1_k is closer to the predicted track $L^2_{k|k-1}$, and therefore the probability of association $L^2_{k|k-1} \rightarrow z^1_k$ is higher than $L^2_{k|k-1} \rightarrow z^2_k$. But actually the measurement z^1_k is generated by track L^1_{k-1} at time step k. Therefore, it is hard to obtain reasonable association when dealing with densely distributed targets only with distance information. Although the target positions in the group are closely spaced, the relative position relationship between the adjacent frames does not change significantly.

Let's take the scenario in Fig. 1 (b) for instance, the predicted tracks can be regarded as three nodes of a triangle and so as the measurements at time step k. The orientation of the 'measurement triangle' barely changes comparing with the 'predicted triangle', since the targets move in formation. For the above reason, we consider introducing the relative position information to improve the association performance.

2.2 Hypergraph Representation of Group Targets Structure

Hypergraph is a generalization of graph. A traditional graph is composed of vertexes and edges, an edge can only connect two vertexes. While the edge of a hypergraph can connect any certain number of vertexes. A hypergraph G can be denoted as $G = (V, E)$, where $V = (v_1, \cdots v_p)$ is the set of vertexes, E is the set of hyperedges those connect a certain number of vertexes. A d-tuple hypergraph is a hypergraph that all hyper-edges of which connects d vertexes. Therefore, a 1-tuple hypergraph is a set of vertexes, a 2-tuple hypergraph is a graph, a 3-tuple hypergraph is a set of triples. The above definition shows that a hypergraph can not only describe the state of the vertexes, but also the interrelationships among the vertexes through the hyperedges. The sketch map of the hypergraph is shown in Fig. 2.

2.3 Hypergraph Matching Probability

With the tracks and measurements represented in hypergraph form, we can resolve the data association by computing the similarity of vertex-vertex or hyperedge-hyperedge.

vertex-vertex. Obviously, it is the traditional way of determining the probability of association based on the distance between two vertexes when tuple number $d = 1$. It is calculated as follows

$$P(v_j, v_i') = \frac{1}{(2\pi)^{n_z/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}x'\Sigma^{-1}x\right) \tag{1}$$

where $x = \left(v_j - v_i'\right)$ is the vector difference between measurement v_j and prediction v_i', Σ is the covariance matrix of x, n_z denotes the number of measurement.

hyperedge-hyperedge. If a feature vector is invariant on adjacent frames, a hyperedge can be instantiated by it. When tracking the targets in a group, the distance between every two targets or the overall group structure can be considered as a constant between adjacent frames. Thus a hyperedge can be instantiated by a vector that connects two vertexes when tuple number $d = 2$. For example, $e_{12} = z_1 - z_2$, $e_{12}' = L^1_{k|k-1} - L^2_{k|k-1}$. A hyperedge can be instantiated by the triangle area that connects three vertexes when tuple number $d = 3$. In this paper, the hypergraph with tuple number $d = 2$ is adopted and the matching probability is calculated as

$$S(e_q, e_p') = \exp\left(-\left|e_q - e_p'\right|\right) \tag{2}$$

where e_q is the q^{th} element of V, e_p' is the p^{th} element of V'.

Compared to the vertex-vertex matching matrix P, the hyperedge-hyperedge matching matrix S naturally contains the relative position information of the group since the hyperedge is instantiated by a vector rather than a particle.

3. Joint Cost Matrix with Hypergraph Matching

In this section, we design the following steps to solve the association problem of group targets tracking, which is blended with hypergraph matching result.

Step 1 Given by the sub-equation of Eq. (28) in [11], the association probability of LMB filter is presented as follows

$$q(z_j; l_i) = \mathcal{N}(z_j; Hm_{l_i}, HP_iH + R) \tag{3}$$

where $1 \leq i \leq I$, $1 \leq j \leq J$, I is the track number at time step $k-1$, J is measurement number at time step k, m_{l_i} and P_i denote the mean position and covariance of the target with label l_i, H denotes the linear observation model, R is the measurement noise covariance. Here let the association matrix be abbreviated as Q,

Step 2 Since the hyperedge matching matrix S indicates the correspondences between hyperedge and hyperedge, it is not the practicable association for vertex to vertex, i.e., prediction to measurement. The vertex-vertex matching matrix X can be resolved according to the relative entropy error method. The specific method can be found in [9].

Step 3 The new joint association matrix is the combination of the traditional association matrix Q and the hypergraph matching matrix X, i.e., $\widetilde{Q} = Q * X^T$, where $*$ denotes dot products. Let $\widetilde{q}(z_j; l_i)$ denotes the element of the i^{th} row and the j^{th} column of the matrix $\widetilde{Q}$. Replace $q_k^{(\xi)}$ with $\widetilde{q}_k^{(\xi)}$ in [11] (26), the joint cost matrix is rewritten as

$$C_{i,j} = -\ln\left(\frac{P_D \sum_{k=1}^{J^{(\xi)}(\ell_i)} w_k^{(\xi)}(\ell_i)\widetilde{q}_k^{(\xi)}(z_j; \ell_i)}{(1-P_D)\kappa(z_j)}\right) \tag{4}$$

where $C_{i,j}$ denotes the cost of the j^{th} measurement associated with the i^{th} track, P_D is detection probability, $w_k^{(\xi)}(\ell_i)$ is the weight of hypothesis $(\theta(\ell_i), \xi)$, $\theta(\ell_i)$ is association solution of target with label l_i, ξ is association map history, $\kappa(z_j)$ denotes the clutter intensity. Then with the ranked assignment and k-th shortest path algorithms [13], the k best hypotheses can be obtained. The remaining steps are the same as the LMB and will not be reiterated here.

4. Numerical Results and Analysis

4.1 Simulation Setup

The following simulation scenario is setup. A group of five targets moves straightly from the upper right to the bottom left in the region $[-20, 20] \times [-15, 15]$, the kinematic state of each target is defined as $[x, \dot{x}, y, \dot{y}]^T$, where $[x, y]^T$ denotes the position and $[\dot{x}, \dot{y}]^T$ denotes the velocity. The motion model and observation model are

$$F_{k|k-1} = \begin{bmatrix} 1 & T_s & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T_s \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{5}$$

The covariance of process noise Q_k and measurement noise R_k are respectively given as

$$Q_k = \sigma_w^2 \begin{bmatrix} T_s^4/4 & T_s^3/2 & 0 & 0 \\ T_s^3/2 & T_s^2 & 0 & 0 \\ 0 & 0 & T_s^4/4 & T_s^3/2 \\ 0 & 0 & T_s^3/2 & T_s^2 \end{bmatrix} \tag{6}$$

$$R_k = \sigma_r^2 \operatorname{diag}(1, 1) \tag{7}$$

where $\sigma_w = 0.01$, $\sigma_r = 0.2$. The sampling period $T_s = 1$s. The clutter number returns per frame subject to Poisson distribution with mean $\lambda = 2$ and uniformly distributed in the surveillance region. In this paper, we only investigate the effect of hypergraph matching on data association. Thus the target survival and detection probability is set to $P_s = 1$ and $P_d = 1$, respectively. The simulation time is 20s.

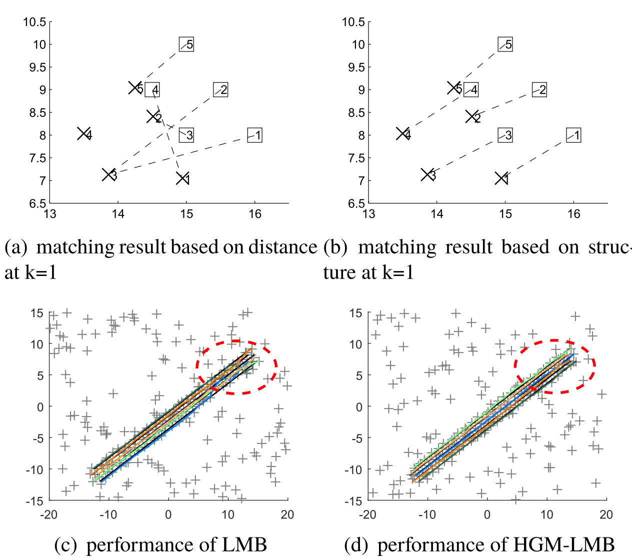
4.2 Result of a Single Simulation

The initial state of each target is set to $[16, -1.1, 8, -0.8]$, $[15.5, -1.1, 9, -0.8]$, $[15, -1.1, 8, -0.8]$, $[14.5, -1.1, 9, -0.8]$, $[15, -1.1, 10, -0.8]$. The survival time of each target lasts for 20s. The single run of the scenario is shown in Fig. 3.

From Fig. 3 (c), it can be seen that the LMB generates a large number of target ID switches during the inception phase, while from Fig. 3 (d) the HGM-LMB filter outputs more clear tracks with less ID switches.

4.3 Results of Monte Carlo Simulations

In this subsection, the tracking performance is measured using the OSPA distance [14] and the 'CLEAR MOT' metrics [15]. Another two filters proposed recently are used as contrast. 500 Monte Carlo simulations are performed and



(a) matching result based on distance at k=1 (b) matching result based on structure at k=1

(c) performance of LMB (d) performance of HGM-LMB

Fig. 3 Association and tracking performance comparison of LMB and HGM-LMB. (a) shows the best path derived from the traditional cost matrix, (b) shows the matching result given by hypergraph matching method, (c) shows the tracks outputted by the LMB filter, (d) shows the tracks outputted by the HGM-LMB. The black lines denote the ground truth of the targets, gray + in (c) and (d) are the measurements, and each colored curve represents a estimated track with unique ID.

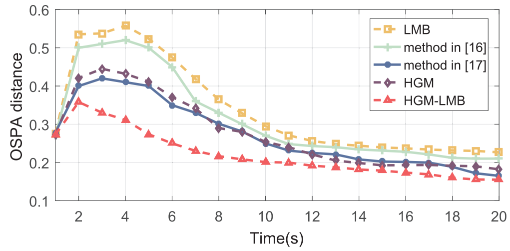


Fig. 4 Mean OSPA distance comparison within 500 Monte Carlo.

Table 1 Evaluation based on CLEAR MOT metrics.

Method \ Indicator	TPR	MOTP	FPR	FNR	ID Switch	MOTA
LMB	83.90%	0.2008	7.77%	7.79%	10.3880	76.13%
method in [16]	84.24%	0.2107	7.59%	7.62%	9.4856	78.65%
method in [17]	90.33%	0.1915	4.78%	5.24%	5.2590	90.24%
HGM	89.18%	0.1964	4.98%	5.28%	4.9150	90.13%
HGM-LMB	**92.65%**	**0.1877**	**3.59%**	**4.17%**	**3.9760**	**89.06%**

the results are given in Fig. 4 and Table 1. The meanings of the ‘CLEAR MOT’ indicator are given as below, and the details of the indicators can be refer to [15].

- TPR (true positive rate): correct association probability;
- MOTP: multiple objects tracking precision;
- FPR (false positive rate): ratio of false positives;
- FNR (false negative rate): ratio of misses;
- IDS: ratio of mismatches, i.e., track ID switches;
- MOTA: multiple objects tracking accuracy.

From Fig. 4, the OSPA value provided by HGM-LMB is lower than three contrast filters. Since [16] focus more on parametric modeling of assumptions about targets interactive motion, while in the tracking scene where the structure is relatively stable and the target interaction is weak, the proposed HGM-LMB is more applicable. The method in [17] also focus on utilizing structure information, which is similar to ours. By borrowing adjacent matrix conception in graph theory, [17] expressed the targets as nodes in a tree. It’s not hard to imagine that a hypergraph is more stable than a tree when describing a graph with an abundance of nodes. For hypergraph matching method, the predicted track cannot match with the corresponding measurement when miss detection occurs, leading to the rupture of the track. While the LMB inherits the idea of the MHT and maintains a hypothesis for the miss detection, which means the track is more possible to be hold to match with the corresponding measurement in the next time step. From Table 1, all the indicators of the HGM-LMB is superior to comparison methods in performance. It further proves the effectiveness of our proposed HGM-LMB in scenes of multi-target cooperative motion.

We now review the relationship between hypergraph matching method and the LMB. Since the distance based association method in LMB cannot cope with closely spaced targets, group structure information has to be taken into consideration. Hypergraph matching is just right for the problem of point matching in two topological graphs and we use it as the association method in HGM-LMB. The hypergraph matching method greatly improves the association performance by using the group structure information, and then improves the trajectories quality.

5. Conclusion

In this paper, we focus on the data association problem in closely spaced multiple targets. We introduce the hypergraph theory to describe the relative position relationship of multiple targets. The hypergraph matching method is used to resolve the association between adjacent frames. We integrate the hypergraph matching result into the LMB filter and then propose the HGM-LMB filter. Through the simulation and analysis, we verify that the relative position is significant information when tracking closely spaced multiple targets and the proposed HGM-LMB filter is superior to the comparison methods in tracking precision and accuracy.

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